NEW ALGORITHMS FOR THE IDENTIFICATION OF A CLASS OF HAMMERSTEIN-WIENER NONLINEAR SYSTEMS

BY

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Dedicated as a humble tribute to

my beloved mother, father, wife, daughters, sons and

to my loving sisters and brothers. Their prayers, sacrifices, inspiration

and love led to this accomplishment

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Nomenclature

Notations and Symbols

t, k	Integer time index
$u(k)$	Input signal
$y(k)$	Output signal
$\eta(k)$	Disturbance / noise signal (i.i.d. random variables)
$g(k)$	Non-linear function
$f(k)$	Non-linear function
$a = (a_1, \cdots, a_p)'$ $b = (b_1, \cdots, b_n)'$	System parameters: linear part
$c = (c_1, \cdots, c_m)'$ $d = (d_1, . \cdots, d_q)'$	System parameters: non-linear part
$\hat{y}(k)$	Predicted/estimated output signal
$\hat{a} = (\hat{a}_1, \cdots, \hat{a}_p)'$ $\hat{b} = (\hat{b}_1, \cdots, \hat{b}_n)'$	Estimate/identified model parameters: Linear part

$\hat{c} = (\hat{c}_1, \cdots, \hat{c}_m)'$
$\hat{d} = (\hat{d}_1, \cdots, \hat{d}_q)'$ Estimate/identified model parameters: non-linear part

μ_i & ζ_i Singular values

ν_i & ξ_i Right singular vectors

σ & δ Left singular vectors.

$e(k)$ White noise (a sequence of independent random variables)

w Column vector of weighting matrix W

W Weighting matrix

q^{-1} Backward shift operator

$\theta(N)$ Parameter vector

$\hat{\theta}(N)$ Estimate of parameter vector

N Number of data points

W Weighting Function

Y_F *Fast Fourier Transform of* $y(k)$

U_F *Fast Fourier Transform of* $u(k)$

F_t *Fast Fourier Transform of* $f(k)$

G_l *Fast Fourier Transform of* $g(k)$

η_F *Fast Fourier Transform of* $\eta(k)$

Abbreviations

NL	Non-linear
ML	Maximum Likelihood
IV	Instrumental Variable
LS	Least Squares
LMI	Linear Matrix Inequality
RLS	Recursive Least Squares
NLS	Nonlinear Least Squares
SVD	Singular Value Decomposition
PRBS	Pseudo Random Binary Sequence
SISO	Single Input Single Output
MIMO	Multi Input Multi Output
GA	Genetic Algorithm
FFT	*Fast Fourier Transform*

PEM	Prediction Error Method
PE	Persistent Exciting
AR	Auto Regressive
MA	Moving Average
ARMA	Auto Regressive Moving Average
TSIA	Two Stage Identification Algorithm [4]
LTI	Linear Time-Invariant
GA	Genetic Algorithm
AI	Artificial Inelegance

THESIS ABSTRACT

Name: **SALEH BIN IBRAHIM AL-RUMAIH**

Title: New Algorithms for the Identification of a Class of Hammerstein-Wiener Nonlinear Systems

Major Field: SYSTEMS ENGINEERING (Automation and Control)

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Identification of non-linear systems is a fundamental problem in several fields. A considerable amount of research has been carried out on modeling, identification and control of non-linear systems. In the study of non-linear systems, usually classes of nonlinear systems such are considered. The purpose of this work is to develop new identification schemes to estimate parameters of non-linear systems of the Hammerstein and the Hammerstein-Wiener structures based on Genetic Algorithms (GA) optimization technique, H_∞ Norm mismatch error, $L_2 - gap$ and $\nu - gap$ metrics.

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تمثل عمليه تحديد الانظمه الغير خطيه ، مشكله اساسيه فى مجالات عدة . لقد تم اجراء العديد من البحوث فى عمليات تحديد المعاملات والتحكم فى النظم الغير خطيه.عادة يتم اعتبار مجموعه من النظم الغير خطيه فى دراسه مجمل النظم الغير خطيه . يهدف هذا البحث إلى تطوير محددات جديدة لكى تقوم بتخمين وتحديد المعاملات للانظمه الغير خطيه كما فى انظمه " هامرشتان ونر" و "همر شتان " و " ونر" , وتقدم هذة الدراسه طرق جديدة بستخدام " الخوارزميات الجينيه " و وماعمل حساب الخط الناتج بستخدام H_∞ و $\nu - gap$ و $L_2 - gap$.

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CHAPTER 1

INTRODUCTION

1.1 History Overview

The last few decades of research showed major theoretical and practical development in the field of control system engineering design. Since World War II, the applications which involved the design of controllers have increased across broad range of systems in industry. These controllers have been successfully used in airplanes, robotics, power systems, oil & gas, chemical, petrochemical, transportation and communication systems. Historically, system identification has been motivated by the need to design better control of dynamic systems. It enables more insight into the system and thus possible to make the system more controllable and observable. Very frequently we are faced with the necessity of experimentally determining some important physical parameters such as heat transfer coefficient, chemical reaction rate, damping factor, and so on. The need for highly accurate system models has been intensified by the development of optimal and adaptive control theories [2].

The mathematical models of dynamic systems are useful in many areas and applications. There are two basic ways of constructing system models:

- *Mathematical Modeling*: This is an analytic approach where we use the basic laws of physics such as heat and mass conservation balance equations and Newton's laws. These equations are used to describe and produce the dynamic behavior of a phenomenon or a process.

- *System Identification*: This is based on some data collected from an experiment or a real-time process. This approach is performed on the system; where the data collected are fitted to a model by assigning suitable numerical values to its parameters.

Identification of unknown systems dynamics has produced a field of research called "system identification". It is the field of modeling dynamic systems from experimental data [3]. In system identification, we define or describe a model structure that behaves similar to an unknown system. Many model structures applicable to linear and nonlinear systems have been developed that have assisted in solving numerous system identification problems.

The bulk of the work done in system identification starts with representing the process as a black box [4]. *A priori* knowledge about the system under identification is critical and can not be overestimated. We must understand the system dynamics. This means that the system structure may not be known in advance, but we must have access to the inputs and outputs. The problem in

system identification is to construct a model which would mimic the *inner mechanism* of the system, using the input/output data. The usual procedure is to select a model structure with some unknown parameters and then to estimate the model parameters. The last step is to check whether the model obtained reasonably explain the input/output data.

The dynamics of linear systems are easily handled today. However, nonlinear systems are still a problem. In fact, all real world systems can said to be nonlinear with different degrees of nonlinearity.

Nonlinear controller design is a very important but a difficult problem in control engineering. Today, most of the available industrial controllers are pure linear controllers due to the difficulties encountered in the nonlinear controller design problem. To build a nonlinear controller we must have enough valid assumptions about the controller structure and the system to be controlled. These assumptions lead to the development of weak control systems since uncertainty effects may not have been taken into account when designing such controllers. The sources of these uncertainties are redundancy or failure in the system components. Robust control theory attempts to deal with these problems to produce a better controller which overcomes these difficulties.

All proposed model structures for system identification contain a set of parameters to identify. These parameters must be adjusted to approximate the real system. This field of research is called “parameter estimation”. Many techniques for parameter estimation have been introduced and used successfully

in different environments. Among those techniques is the Least-Square Estimation (LSE). LSE has been used to solve the parameter estimation problem for linear systems and linearized nonlinear systems.

The main difficulty in handling the parameter estimation and system identification problems arises when the proposed model structures are not good enough to handle the large changes in system dynamics and enough of *priori* knowledge does not exist. This occurs in the case of nonlinear systems. In nonlinear systems, traditional theories will not perform successfully if the system cannot be linearized or if noise is present. In these cases, the results of the system identification process might be so poor that the required goals are not achieved.

1.2 Nonlinear system identification

In the last decades, many research activities have been carried out on modeling, identification and controller design of nonlinear systems [5, 6, 7]. The reason is that nonlinear models describe most of the physical processes and naturally the use of nonlinear analysis increases accuracy and performance of the system.

Most dynamical systems can be better represented by nonlinear models, which are able to describe the global behavior of the system over the whole operating range, rather than by linear ones that are only able to approximate the

system around a given operating point. A well known method that can represent a wide class of nonlinear systems is the well known Volterra series expansion [5]. However, other frequently studied classes of nonlinear models are the so called block-oriented models, which consist of the interconnection of Linear Time-Invariant (LTI) systems and static nonlinearities. The most commonly used nonlinear block oriented models structures are:

- **Hammerstein model,** which consists of the cascade connection of a static nonlinearity followed by a LTI system (see Figure 2.1)
- **Wiener model**, in which the order of the linear and the nonlinear blocks in the cascade connection is reversed (see Figure 2.2)
- **Wiener-Hammerstein Model**, which consist of a linear system G_1 in cascade with a static nonlinear element N and a linear system G_2. Many chemical and other industrial processes are of this type (see Figure 2.3)
- **Hammerstein-Wiener Model,** which consist of two static nonlinear elements $N_1(.)$ and $N_2(.)$ surrounding a linear block G(z) (see Figure 2.4).

These models have been successfully used in representing nonlinear systems in a number of practical applications in the areas of chemical processes [5], biological processes [8], signal processing [9], communications, and control [10]. Several techniques have been proposed in the literature for the identification of Hammerstein and Wiener models (see for instance [1, 5, 8, 10-19], and the references therein). Among them three main approaches can be distinguished.

The first one is the traditional iterative Algorithm proposed by Narendra and Gallman [9]. In this Algorithm, an appropriate parameterization of the system allows the prediction error to be separately linear in each set of parameters characterizing the linear and the nonlinear parts. The estimation is then carried out by minimizing alternatively with respect to each set of parameters, a quadratic criterion on the prediction errors. An analytical counter example by Stoica [3] showed that the original Algorithm could be divergent in some particular cases. A second approach**,** based on correlation techniques, is introduced in [15-18]. This method relies on a separation principle, but with the rather restrictive requirement on the input to be white noise. A more recent approach for the identification of Hammerstein-Wiener systems has been introduced by Bai [1]. This Algorithm is based on least squares estimation (LSE) and singular value decomposition (SVD), however it only applies to the single-input/single-output (SISO) case, and consistency of the estimates can only be assured for the case of the disturbances being white noise, or in the noise-free case.[1]

One approach to understand the nonlinear behavior is to form a mathematical model of the process. To achieve this, a mathematical model of each unit operation has to be formed by making some simplifying assumptions, and then these models are combined to obtain a model representing the complete system.

The Hammerstein and Wiener models are used to model several classes of nonlinear systems. Their flexibility lies in having the nonlinearity entirely separate from the common and easily realizable linear parts. Identification gives more insight into the system and makes the system more controllable and observable.

This thesis proposes new methods for handling the system identification. Our tools in doing this is based on basic knowledge of traditional system identification and control theories as well as:

- Artificial Intelligent (Al) approach using genetic algorithms (GAs).
- H_∞ Norm of the Mismatch error
- L_2 gap metric
- ν gap metric

We found GAs to be a very useful tool that can significantly help in providing new solutions for system identification. Our research effort starts by exploring the advantages of Genetic Algorithms (GAs) in solving the parameter estimation problem for nonlinear systems. Then, in the frequency domain we have used the H_∞ mismatch error between the estimated model and the simulated system to identify/estimate the model (Hammerstein system) parameters. This method has been expanded to perform the parameter estimation/identification of the Bai's System [1].

Then, we proposed a new frequency domain based hybrid methodology which combines L_2 gap error, Least-Square Estimation and Singular Value Decomposition (SVD). This Algorithm has been implemented on the parametric identification of the Bai's system.

The ν-gap metric has been explored to see how it can be used for the identification of the Hammerstein class of non-linear systems as well as the Bai's system.

Our focus in this thesis is nonlinear dynamic systems. Parameter estimation for nonlinear systems in uncertain or noisy environments is a difficult problem. The main cause of difficulty is that most currently available parameter estimation techniques are affected by noise.

Typically used model structures in system identification are Moving Average (MA) and Auto-Regressive Moving Average (ARMA) models. The structure of these models depends on the type of applications. Parameter estimation techniques that have typically been used in the past include Least Square (LS), Maximum Likelihood (ML), and Instrumental Variable (IV). Although these techniques very often provide good results with respect to convergence and error minimization, they are limited in their ability to handle multi-modal error surfaces and the presence of noise.

1.3 Contribution of the Thesis

The thesis objectives are:

1. The Genetic Algorithms is used as an optimization tool to identify the parameters of the Hammerstein-Wiener non-linear systems. The equivalence between the Two-Stage Algorithm [1] developed in the literature for the identification of these systems is used to formulate the problem as an optimization problem to be solved using GA's. Some examples are presented to illustrate the new approach and its performance. The performance of the GA identification-based results is compared with that of Bai's two stage Algorithm [1].

2. An H_∞-Norm based scheme is developed for Hammerstein system. This scheme is expanded to identify the Bai's system [1]. The H_∞-Norm scheme is based on the work done by Al-Amer and Al-Sunni [11].

3. The L_2-gap of the mismatch error based algorithm is used to perform the parametric identification of the Bai's system. This work is based on the work done by Al-Amer and Al-Sunni [12, 20]

4. The ν-gap metric has been explored for the identification of the Hammerstein class of non-linear systems as well as the Bai's system.

1.4 Organization of the Thesis

The research effort in this thesis is focused on the development of new methodologies and algorithms that significantly help in solving the system identification for block oriented non-linear systems which include Hammerstein, Hammerstein-Wiener, and Bai[1] systems.

The first chapter of this thesis is used to go over the history of nonlinear system identification, introduce and define the problem, state the thesis objectives and outline the organization.

The second chapter covers the selected literatures in the area of nonlinear system identification of Hammerstein-Wiener, GA, Bai's Two Stage Algorithm [1], H_∞ mismatch error, L_2-gap and ν-gap metrics based identification schemes. Also a review of some required mathematical tools are covered.

In Chapter 3, we describe the motivational background and related work to system identification of nonlinear systems using Genetic Algorithms (GA). After that, we present the Bai's [1] nonlinear system that is our subject of identification using GA. Then, we describe a few applications/examples that illustrate the strength and powerful abilities of GAs in handling difficult nonlinear systems design problems.

In Chapter 4, we consider Identification of Hammerstein Models to minimize the H_∞ Norm of the Mismatch Error. This work is a modified algorithm of the work presented in [11]**.** This modified algorithm is used to identify Hammerstein

models. Hammerstein Models are special in the sense that they can be transformed into linear models and linear systems techniques may be applicable. A brief introduction of the subject is covered that is followed by stating the problem. Then the proposed iterative identification algorithm is presented to obtain a model that minimizes the H_∞ norm of the mismatch error. Illustrative examples are given with some concluding remarks.

In Chapter 5, we expand the work done in Chapter 4 to perform parametric identification of the Bai's system [1]. This work involves the use of H_∞ Norm of the mismatch error between the true plant and the identified model. Then the proposed iterative identification algorithm is presented to obtain a model that minimizes the H_∞ norm of the mismatch error. Illustrative examples are given with some concluding remarks.

In Chapter 6, we consider Identification of Hammerstein Models to minimize the L_2-gap error. The work done in [12, 20] is modified and extended to perform parametric identification of the Bai's system [4]. This work involves the use of the L_2-gap between the true system and the identified model of Bai's system. An introduction is presented to the use of gap metrics as a measure of error that enables one to obtain nominal models and bounds on the uncertainty that are essential for robust controller design. The gap metric was introduced to measure the closeness of two systems having the same feedback configurations. It is well known that two closed loop systems can be very closed even if the norm of the

difference of the open loop blocks is arbitrarily high. Then the proposed iterative identification algorithm is presented to obtain a model that minimizes the L_2 gap error. Illustrative examples are given.

In Chapter 7, we explore the use of the ν-gap metric to identify the Hammerstein systems as well as the Bai's system [1]. The ν-gap metric has well established properties that are useful in the study of robust control systems. Properties of ν-gap are extensively studied by Vinnicombe [21].] Date and Vinnicombe [22] have proposed an algorithm for the identification of linear SISO systems using the ν-gap metric. The algorithm involves solving a series of LMI optimization problems followed by Hankel approximation. Al-Amer [12], proposed an iterative weighted least squares algorithm to solve the ν-gap metric identification problem for linear systems. The use of ν-gap for measuring uncertainty in nonlinear system was presented in [21].

In Chapter 8, we analyze the performance of the GA, H_∞ Norm of the mismatch error, and L_2 gap error, and compare their results and make some conclusions about their performances.

Finally, Chapter 9 contains our conclusions and outlines of suggested future work. We hope that this thesis will initiate a change in the current way of thinking and handling the relationships between traditional and intelligent system identification and frequency based iterative identification algorithms. It is important to fill the gap between traditional and intelligent control.

CHAPTER 2

LITERATURE BACKGROUND

2.1 Introduction

Most physical systems have nonlinear characteristics outside a limited linear range. A well-known method that can represent a wide class of nonlinear systems is the Volterra series expansion. In some particular cases, nonlinear systems can be separated into block of linear subsystems and zero-memory nonlinearties in various combinations.

One approach to understanding the nonlinear behavior is to form a mathematical model of the process. To achieve this, a mathematical model of each unit operation has to be formed by making some simplifying assumptions, and then these models are combined to obtain a model representing the complete system.

The Hammerstein and Wiener models are used to model several classes of nonlinear systems. Their flexibility lies in having the nonlinearity entirely separate

from the common and easily realizable linear parts. Identification gives more insight into the system and makes the system more controllable and observable.

2.2 Input Signals

The input signal used in an identification experiment can have a significant influence on the resulting parameter estimations. The following signals are the most commonly used for parameters estimation/identification:

- Step Function
- Pseudorandom Binary Sequence
- Autoregressive Moving Average Process
- Sum of Sinusoids

In this work, we are mainly working with the Sum of Sinusoids as our inputs to the systems under study because we can assure ourselves that we are going to have an input signal with enough PE to estimate all system parameters. More details can be found in [3]

2.2.1 Sum of Sinusoids

In this class of input signal, *u(k)* is given by:

$$u(k) = \sum_{j=1}^{m} a_j \sin(\omega_j k + \varphi_j) \qquad (2.1)$$

where the angular frequencies ω_j are distinct.

$$0 \le \omega_1 < \omega_2 < \cdots < \omega_m \le \pi \qquad (2.2)$$

For a sum of sinusoids the user has to choose the amplitudes { a_j }, the frequencies { ω_j } and the phases { φ_j }. Figure 2.1 illustrates a typical example for the input signal using a sum of sinusoids [3].

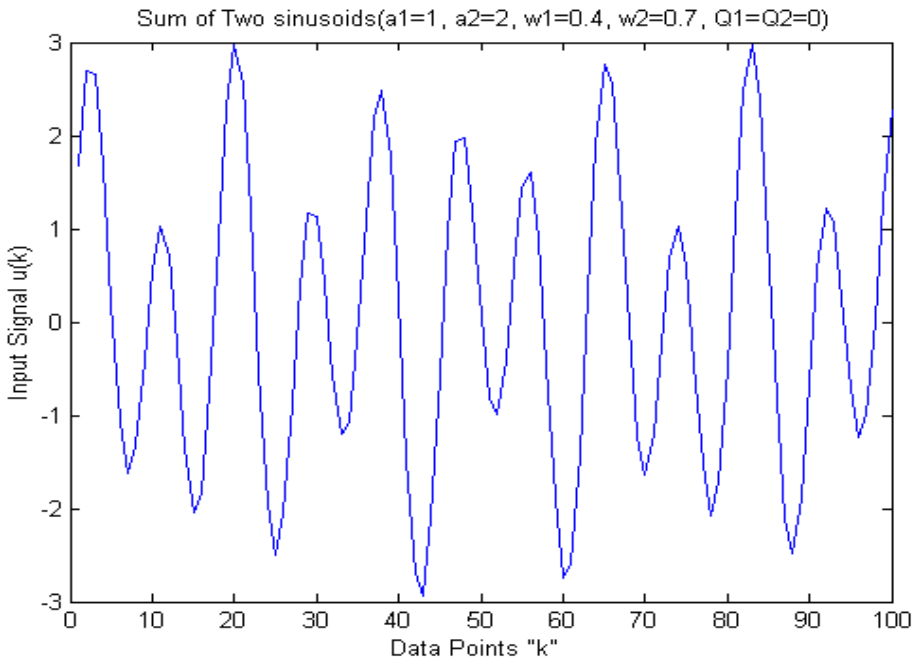

Figure 2.1: A sum of two sinusoids

2.3 Persistent Excitation

For linear systems, a signal u(k) is said to be persistently exciting (PE) of order *n* if the following conditions are satisfied:

$$\text{(i)} \qquad r_u(\tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} u(t+\tau)u(t); \qquad \tau = 0,1,2,\cdots \tag{2.3}$$

and

$$\text{(ii)} \qquad \text{the matrix } R_u(n) = \begin{pmatrix} r_u(0) & r_u(1) & \cdots & r_u(n-1) \\ r_u(1) & r_u(0) & & \vdots \\ \vdots & & \ddots & \vdots \\ r_u(n-1) & \cdots & \cdots & r_u(0) \end{pmatrix} \tag{2.4}$$

is positive definite [3]

The following two properties are applicable to our work which are stated without proof.

Property 1[3]

Let u(t) be a multivariable ergodic process of dimension nu. Assume that its spectral density matrix is positive definite in at least n distinct frequencies (within the interval $(-\pi, \pi)$ *). Then u(t) is persistently exciting of order n.*

Property 2[3]

Let u(t) be a scalar signal that is persistently exciting of order n, then its spectral density is nonzero in at least n frequencies.

The spectral density for the sum of sins (equation (2.1)) is given as follows:

$$\phi_u(\omega) = \sum_{j=1}^{m} \frac{C_j}{2}[\delta(\omega - \omega_j) + \delta(\omega + \omega_j)] \qquad (2.5)$$

thus the spectral density is nonzero(in the interval $(-\pi, \pi)$ in exactly n points, where

$$n = \begin{cases} 2m & if\ 0 < \omega_1,\ \omega_m < \pi \\ 2m-1 & if\ 0 = \omega_1\ or\ \omega_m = \pi \\ 2m-2 & if\ 0 = \omega_1\ and\ \omega_m = \pi \end{cases} \qquad (2.6)$$

It follows from the properties 1 and 2 above that u(t) is PE of order n as given by equation (2.6)

2.4 Singular Value Decomposition

Let ***A*** be a general real (***m*** X ***n***) matrix.

The **singular value decomposition (SVD)** of A is the factorization $A = USV^T$,where U is an (*m* X *m*) orthogonal matrix, V is an (*n* X *n*) orthogonal

matrix and S is an (*m* X *n*) diagonal matrix with real, non-negative elements that has the following forms:

$$S = \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} \quad if \quad m \geq n \quad and \quad S = \begin{bmatrix} \Sigma & 0 \end{bmatrix} \quad if \quad m < n,$$

where $\Sigma = diag(\sigma_1, \cdots\cdots\cdots, \sigma_r)$ *with* $r = \min(n, m)$.

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \geq 0$$

The σ_i are called the **singular values** of A, and the first ***r*** columns of V are the **right singular vectors** and the first ***r*** columns of U are the **left singular vectors**.

We assume now $m \geq n$. If $r = rank(A) < n$, then

$$\sigma_1 \geq \sigma_2 \geq \cdots\cdots \geq \sigma_r > \sigma_{r+1} = \cdots\cdots = \sigma_n = 0$$

If $\sigma_r \neq 0 \quad \sigma_{r+1} = \cdots\cdots = \sigma_n = 0$, then r is the rank of A. **SVD** can thus be used for rank determination.

If A is complex, then its SVD is $A = U\Sigma V^H$ where U and V are unitary, and Σ is as before a matrix with a real non-negative diagonal elements.

The SVD provides a numerically robust solution to the least squares problem. The solution

$$x = (A^T A)^{-1} A^T b$$

becomes with $A = USV^T$:

$$x = VS^{-1}U^T b$$

The singular value decomposition (SVD) is a powerful technique in many matrix computations and analysis. Using the SVD of a matrix in computations, rather than the original matrix, has the advantage of being more robust to numerical error. The SVD is employed in a variety of applications, from least square problems to solving systems of linear equations. Each of these applications exploits key properties of the SVD, its relation to the rank of a matrix and its ability to approximate matrices of a given rank. Many fundamental aspects of linear algebra rely on determining the rank of a matrix, making the SVD an important and widely used technique.

2.5 Hammerstein Model

The Hammerstein model consists of the cascade connection of a static (zero-memory) nonlinearity followed by a linear time-invariant system [4].

The static nonlinear element scales the input $u(t)$ and transforms it to $x(t)$, and the dynamics are modeled by a linear transfer function, whose output is *y(t).* The Hammerstein model can be described by the following equations.

$$y(k)+a_1y(k\text{-}1)\ +\cdots+a_n\ y(k\text{-}n)\ \ =b_0+b_1x(k\text{-}1)+b_2x(k\text{-}2)+\cdots+b_mx(k\text{-}m) \quad (2.7)$$

$$x(k)\ =\ f(u(k))=\sum_{i=1}^{m}c_i g_i(u(k)) \quad (2.8)$$

g_i are know functions. Example polynomial $g_i(u(k))=x^i$

Where $\mathrm{u(k)}$ is the input to the system, $\mathrm{y(k)}$ is the output of the system and $\mathrm{x(k)}$ is the nonlinear function of the input. $\mathrm{x(k)}$ cannot be measured, but it can be eliminated from the equations. The above equation can be written in the following form in which the intermediate variable $\mathrm{x(k)}$ has been removed.

$$y(k)\ =\frac{B(q^{-1})}{A(q^{-1})}f(u(k)) \quad (2.9)$$

where the polynomial $A(q^{-1})$ and $B(q^{-1})$ are

$$A(q^{-1})=1+a_1q^{-1}+a_2q^{-2}+\ldots+a_nq^{-n} \quad (2.10)$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} + \ldots + b_m q^{-m} \quad (2.11)$$

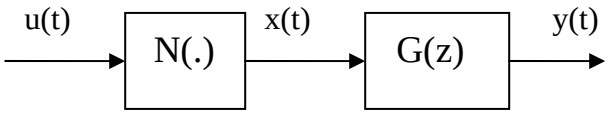

Figure 2.2: Block diagram of The Hammerstein Model

2.6 Wiener Model

The Wiener model of a nonlinear system is constructed by a nonlinear gain cascaded after a linear sub-system. The linear dynamic part of the Wiener model is given by

$$x(k) = \frac{B(q^{-1})}{A(q^{-1})} u(k) \quad (2.12)$$

where $x(k)$ is the input to the non-linearity, and $u(k)$ is the input to the system. The polynomials $A(q^{-1})$ and $B(q^{-1})$ are as defined in equations (2.10) and (2.11). The observed output $y(k)$ of the system is

$$y(k) = f(x(k)) = \sum_{i=1}^{p} c_i g_i(x(k)) \quad (2.13)$$

A typical choice of g_i are polynomials $g_i(x) = x^i$

Thus, the problem of identification of the Hammerstein and Wiener models is to estimate the coefficients of the linear part $[a_1, a_2, ..., a_n, b_1, ..., b_m]$ and the parameters of the nonlinear part $[c_1, c_2, ..., c_p]$, from input-output data.

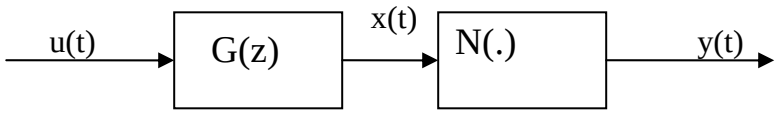

Figure 2.3: Block diagram of The Wiener Model

2.7 Wiener-Hammerstein Model

Figure 2.4 is a block diagram of the well-known Wiener-Hammerstein model or the G-model. It consists of a linear system G_1 (with impulse response $h_1(t)$ in cascade with a static zero-memory nonlinear element N and a linear system G_2 (with impulse response $h_2(t)$). Many chemical and other industrial processes are of this type [10].

For a SISO system, the Box-Jenkins model of the Wiener-Hammerstein nonlinear system shown in Figure 2.4 can be represented as:

$$
\begin{aligned}
y(t) &= \frac{D(q^{-1})}{C(q^{-1})}\sum_{i=1}^{p} c_i g_i\left[\frac{B(q^{-1})}{A(q^{-1})}u(k)\right] \\
&= \frac{D(q^{-1})}{C(q^{-1})}\; N\left[\frac{B(q^{-1})}{A(q^{-1})}u(k)\right] \\
&= \frac{D(q^{-1})}{C(q^{-1})}\; N\left[w(k)\right] \\
&= \frac{D(q^{-1})}{C(q^{-1})}\; x(k)
\end{aligned}
\qquad (2.14)
$$

Where $g_i(.)$ are the nonlinear functions used to describe the nonlinear zero memory subsystem $N(.)$ of order *p* and *k* is an integer time index. A typical choice for the functions $g_i(.)$ is $g_i(w(k)) = w^i(k)$, in which case the nonlinear subsystem is represented by a polynomial expansion. The pair of polynomials $\left(D(q^{-1}), C(q^{-1})\right)$ and $\left(B(q^{-1}), A(q^{-1})\right)$ are related to the Linear Time-Invariant blocks $G_2(z)$ and $G_1(z)$ respectively.

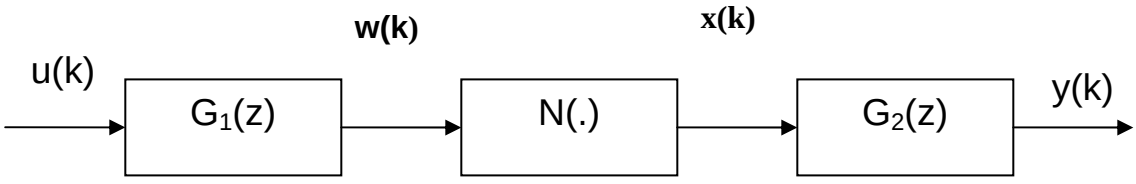

Figure 2.4: Block diagram of The Wiener- Hammerstein Model

2.8 Hammerstein-Wiener Model

Hammerstein-Wiener models may be considered as the system where two static nonlinear elements $N_1(.)$ and $N_2(.)$ surround a linear block $G(z)$ (Figure 2.5).

Here the model is given as:

$$\begin{aligned} y(t) &= \sum_{j=1}^{q} d_j f_j \left[\frac{B(q^{-1})}{A(q^{-1})} \sum_{i=1}^{p} c_i g_i [u(t)] \right] \\ &= \sum_{j=1}^{q} d_j f_j \left[\frac{B(q^{-1})}{A(q^{-1})} N_1[u(t)] \right] \\ &= \sum_{j=1}^{q} d_j f_j [w(t)] \\ &= N_2[x(t)] \end{aligned} \qquad (2.15)$$

Where $(g_i,\ i=1,2,\cdots,p)$ and $(f_j,\ j=1,2,\cdots,q)$ are the nonlinear functions for the nonlinear blocks N_1 and N_2, respectively.

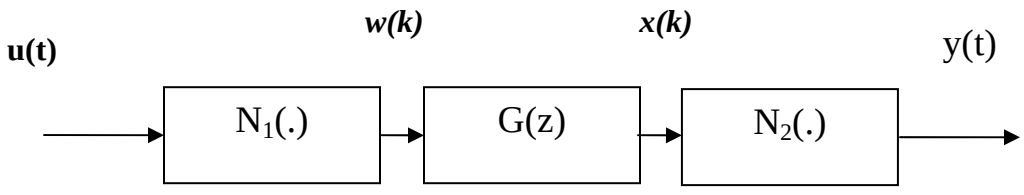

Figure 2.5: Block diagram of The Hammerstein-Wiener Model

2.9 Bai's Identification Scheme

A class of representation for the type described in section 2.8 of nonlinear systems is given by Bai [1], where a discrete-time nonlinear dynamic system is considered as shown in equation (2.16) below.

$$y(k)=\sum_{i=1}^{p}a_i\left\{\sum_{l=1}^{q}d_l\ g_l\left[y(k-i)\right]\right\}+\sum_{j=1}^{n}b_j\left\{\sum_{t=1}^{m}c_t\ f_t\left[u(k-j)\right]\right\}+\eta(k) \qquad (2.16)$$

In equation (2.16), *y(k)* , *u(k)* and $\eta(k)$ are the system output, input and noise at time *k* respectively. The $\left(g_i,\ i=1,2,\cdots,p\right)$ and $\left(f_j,\ j=1,2,\cdots,q\right)$ are non-linear functions and

$$a^T=\left[\mathrm{a}_1,\mathrm{a}_2,...,\mathrm{a}_\mathrm{p}\right],\ b^T=\left[\mathrm{b}_1,\mathrm{b}_2,...,\mathrm{b}_\mathrm{n}\right],\ c^T=\left[\mathrm{c}_1,\mathrm{c}_2,...,\mathrm{c}_\mathrm{m}\right],\ d^T=\left[\mathrm{d}_1,\mathrm{d}_2,...,\mathrm{d}_\mathrm{q}\right]$$

denote the system parameter vectors. The model shown in equation (2.16), may be considered as the system where two static nonlinear elements N_1 and N_2 surround a third block representing a ratio of linear sums of non-linear functions of *u,* and *y*. This system model is shown in figure 2.6.

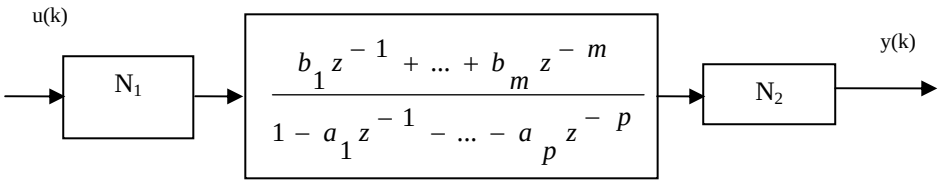

Figure 2.6: Block diagram of The Bai's System Model

The identification of nonlinear systems such as the one presented above is a topic, which has received considerable attention over the last two decades (see for example [5, 6, 23]). The purpose of identification is to estimate the parameters of the system. In the case of the Bai's system, for example, we need to estimate the unknown parameter vectors *a, b, c* and *d* of equation (2.16), from the observed input-output measurements. The functions $\left(f_j,\ j=1,2,\cdots q\right)$ and $\left(g_i,\ i=1,2,\cdots p\right)$ are assumed to be a priori known smooth functions and the orders *q, n, p* and *m* are assumed to be known as well.

Bai [1] presented an optimal two stage identification algorithm for the Hammerstein-Wiener nonlinear system given in equation (2.16). He has shown that his algorithm converges to the true parameters with no noise or white noise:

Bai's Uniqueness Assumption: Consider the system in equation (2.16):

- Assume that Θ_{ad} and Θ_{bc} are not both zero.
- Assume that $\|a\|_2 = 1 \ \ and \ \ \|b\|_2 = 1$ and the signs of the first non-zero elements of *a* and *b* are positive.

The Algorithm consists of two steps: The first one is the Least Squares estimates of the products of parameters and the second one is the Singular Value Decomposition (SVD) of two matrices to extract the estimates of individual parameters.

In the following, we present the major steps in the optimal two-stage algorithm.

Define:

$$\theta = \left(b_1c_1, \cdots, b_1c_m, \cdots, a_pd_1, \cdots, a_pd_q \right)^T \tag{2.17}$$

Let,

$$\Theta_{bc} = \begin{pmatrix} b_1c_1 & b_1c_2 & \cdots & \cdots & b_1c_m \\ b_2c_1 & b_2c_2 & & & b_2c_m \\ & \cdots & \cdots & \cdots & \\ \\ b_nc_1 & b_nc_2 & \cdots & \cdots & b_nc_m \end{pmatrix} \tag{2.18}$$

$$\Theta_{ad} = \begin{pmatrix} a_1d_1 & a_1d_2 & \cdots & \cdots & a_1d_q \\ a_2d_1 & a_2d_2 & & & a_2d_q \\ & \cdots & \cdots & \cdots & \\ \\ a_pd_1 & a_pd_2 & \cdots & \cdots & a_pd_q \end{pmatrix} \tag{2.19}$$

and the regressor vector φ

$$\varphi = \left(f_1[u(k-1)], \cdots, f_m[u(k-1)], \cdots, f_1[u(k-n)], \cdots, g_1[y(k-p)], \cdots, g_q[y(k-p)\right)^T$$

(2.20)

The parametric model of the system in equation (2.16) can now be written as [24]:

$$y(k) = \varphi^T(k)\theta + \eta(k)$$

For a given data set $\{u(k), y(k)\}_{k=1}^{N}$, let

$$Y_N = (y(1), \cdots, y(N))', \ \eta_N = (\eta(1), \cdots, \eta(N))',$$

$$\Phi_N = (\phi^T(1), \cdots, \phi^T(N))$$

Then,

$$Y_N = \Phi_N \theta + \eta_N \tag{2.21}$$

Further let

$$\hat{\theta}(N) = (\hat{\theta}_1, \cdots, \hat{\theta}_{nm}; \hat{\theta}_{nm+1}, \cdots . \hat{\theta}_{nm+pq})^T$$

$$\begin{aligned} &\hat{a}(N) = (\hat{a}_1(1), \hat{a}_1(1), \cdots, \hat{a}_p(1))^T, \quad \hat{b}(N) = (\hat{b}_1(1), \hat{b}_1(1), \cdots, \hat{b}_n(1))^T \\ &\hat{c}(N) = (\hat{c}_1(1), \hat{c}_1(1), \cdots, \hat{c}_m(1))^T, \quad \hat{d}(N) = (\hat{d}_1(1), \hat{d}_1(1), \cdots, \hat{d}_q(1))^T \end{aligned} \tag{2.22}$$

Denote the estimates of θ, a, b, c, d respectively using the set $\{u(k), y(k)\}_{k=1}^{N}$ and

$$\hat{\Theta}_{ad}(N) = \begin{pmatrix} \hat{\theta}_{nm+1} & \cdots & \hat{\theta}_{nm+q} \\ \hat{\theta}_{nm+q+1} & \cdots & \hat{\theta}_{nm+2q} \\ & \ddots & \\ \hat{\theta}_{nm+(p-1)q+1} & & \hat{\theta}_{nm+pq} \end{pmatrix} \tag{2.23}$$

$$\hat{\Theta}_{bc}(N) = \begin{pmatrix} \hat{\theta}_1 & \cdots & \hat{\theta}_m \\ \hat{\theta}_{m+1} & \cdots & \hat{\theta}_{2m} \\ & \ddots & \\ \hat{\theta}_{(n-1)m+1} & & \hat{\theta}_{nm} \end{pmatrix} \tag{2.24}$$

denotes the estimate of $\Theta_{bc} \quad and \quad \Theta_{ad}$ respectively.

The Identification Algorithm given in [1] can be summarized as follows:

Step 1: Calculate the least squares estimate of the vector in equation (2.21).

$$\hat{\theta}(N) = \hat{\theta}_{ls}(N) = \left(\Phi_N^T \Phi_N\right)^{-1} \Phi_N^T Y_N \tag{2.25}$$

Step 2: Obtain $\hat{\Theta}_{bc}(N)$ and $\hat{\Theta}_{ad}(N)$ from equations (2.23) and (2.24), such that

$$\hat{\theta}_{bc}(N) = \sum_{i=1}^{\min(m,n)} \sigma_i \ \mu_i \ \nu^T{}_i \qquad \hat{\theta}_{ad}(N) = \sum_{i=1}^{\min(p,q)} \delta_i \ \xi_i \ \zeta^T{}_i \tag{2.26}$$

Where $\mu_1,...,\mu_n$, $\nu_1,...,\nu_m$, $\xi_1,...,\xi_p$, $\zeta_1,...,\zeta_q$ are n, m, p, q-dimensional orthonormal vectors and $\sigma_1,...,\sigma_{\min(n,m)}$ and $\delta_1,...,\delta_{\min(p,q)}$ are the nonzero singular values of $\hat{\Theta}_{bc}(N)$ and $\hat{\Theta}_{ad}(N)$ respectively. Thus using the above SVD, the parameter estimates may be found according to

$$\hat{b}(N) = s_\mu \ \mu_1 , \ \hat{c}(N) = s_\mu \ \sigma_1 \ \nu_1 , \ \hat{a}(N) = s_\xi \ \xi_1 , \ \hat{d}(N) = s_\xi \ \delta_1 \ \zeta_1$$

where s_μ and s_ξ are the signs of the first non-zero elements of μ_1 and ξ_1 respectively.

2.10 L_2 -gap and the ν -gap metrics

The use of ν -gap or l_∞ as a measure of the error enables one to obtain nominal models and bounds on the uncertainty that are essential for robust controller design. The gap metric was introduced to measure the closeness of two systems having the same feedback configurations. It is well known that two closed loop systems can be very close even if the norm of the difference of the open loop blocks is arbitrarily high. For systems under feedback, it may be more appropriate to use the gap metric to measure the error. El-Sakkary [25] have proposed the following gap metric for SISO systems

$$\delta_L(P_1,P_2)=\sup_{\omega\in\Omega}\frac{\left|P_1(j\omega)-P_2(j\omega)\right|}{\sqrt{1+\left|P_1(j\omega)\right|^2}\sqrt{1+\left|P_2(j\omega)\right|^2}} \tag{2.27}$$

This is now known as the L_2 -gap metric.

A closely related gap metric is the *v*-gap metric which is defined as [35].

$$\delta_\nu(P_1,P_2)=\begin{cases}\delta_L(P_1,P_2) & \text{if } 1+P_2^*(j\omega)P_1(j\omega)\neq 0 \ \forall\omega \text{ and} \\ & wno(1+P_2^*P_1)+\eta(P_1)-\eta(P_2)=0 \\ 1 & \text{otherwise}\end{cases} \tag{2.28}$$

where $\eta(P)$ and *wno(P)* denote the number of right half plane poles and winding number of *P* respectively.

The ν-gap metric has well established properties that are useful in the study of robust control systems. Properties of ν-gap are extensively studied by Vinnicombe [26]. In [22], an algorithm was proposed for the identification of linear SISO systems using the ν-gap metric. The algorithm involves solving a series of LMI optimization problems followed by Hankel approximation. Al-Amer [12}, proposed an iterative weighted least squares algorithm to solve the L_2-gap metric identification problem for linear systems. The use of ν-gap for measuring uncertainty in nonlinear system was presented in [27].

2.11 Literature Review

The class of systems we are addressing may be classified further to three categories: Hammerstein, Wiener, and Wiener-Hammerstein systems. In the documentation of the literature review, the literature review is presented in according to this classification and in the order they appear above.

Many identification methods have been developed to identify the *Hammerstein model;* as in Figure 2.1. All of the techniques largely depend on the prior knowledge of the system. Narendra and Gallman [9] for the first time in 1966 used an iterative method for the identification of Hammerstein model with a

polynomial nonlinearity. The method utilizes the alternate adjustment of the parameters of the linear and NL parts of the model while minimizing the error function. In 1971 Chang and Luus [16] used a non-iterative technique for the identification of a Hammerstein system where the transfer function may have zeros. Zhu and Control [28] proposed a relaxation iteration scheme by making use of a model structure in which the error is bilinear in the parameters. The order of the linear part and NL part are determined by looking at an output error related criterion, which is control relevant. Rangan et al. Billings and Fakhouri [8] presented an identification algorithm for the systems having the structure of a Hammerstein model using the cross-correlation techniques to de-couple the identification of the linear dynamics from the characterization of the nonlinear element. Li [29] demonstrated that both Genetic Algorithm (GA) and the method of approximating nonlinearity with piecewise linear systems can be used to estimate a Hammerstein model. The algorithm has been proven to be robust, globally stable and powerful. Kristinsson and Dumont [30] demonstrated the usefulness of GA to estimate both continuous and discrete time systems and for identifying poles and zeros or physical parameters of a system. Hatanaka and Uosaki [31] proposed a novel approach for identification using the Genetic Programming and determined the nonlinearity of the system. Tobin et al. [32] used a point-slope parameterization of the static nonlinearity that leads to a computationally tractable optimization problem. The identification method considered, using a Hammerstein feedback model with piecewise linear static

maps simultaneously approximates the linear dynamic and static nonlinear blocks of the nonlinear feedback model. Eskinat and Johnson [4] used the identification methods for simulated distillation columns and to an experimental heat exchanger process. Boutayeb et al [18] have given an algorithm which transforms the nonlinear model into a model which is linear in parameters and then they derived the pseudo-inverse technique, leading to a consistent estimator or the initial realization as well the model of the noise. Al-Amer and AL-Sunni [11] proposed a new iterative procedure to identify Hammerstein models. The algorithm minimizes the infinity norm of the deviation between the true model and identified model.

The Wiener model, as in Figure 2.2, is constructed by a nonlinear gain cascaded after a linear system. Zhu and Control [28] studied the identification of SISO Wiener model. They extended the *assymetic* ASYM method which provides the systematic solutions to the problems of identification for both open loop and closed-loop operation. While the parameter estimation, the bilinear-in-the-parameters property of the high order model is used to derive relaxation algorithm. Wingren [27] derived a recursive prediction error identification algorithm for the Wiener model. Where the linear dynamic block is modeled as a SISO transfer function operator and the static nonlinearity is approximated with a piecewise linear function. Kalafatis [33] proposed an approach for the identification of Wiener systems in a noisy environment. The estimated models are represented in terms of the frequency response of the linear subsystem and

the inverse of the static nonlinearity. Chou and Verhaegen [17] presented a method to identify Wiener models with a general disturbance configuration in closed-loop using the indirect approach. They used a disturbance structure such that the noise enters both at the output side and in between the LTI and NL block. Hu and Wang [34] identified the impulse response functions of the linear part and the polynomial coefficients of the nonlinear part of the discrete-time Wiener model using the three-lever-pseudorandom-sequences with different amplitudes as input signals.

Very few efforts have been made to study the Wiener-Hammerstein model (Figure 2.3) or the G-model when compared to the work done for the Hammerstein and the Wiener models [5]. Billings and Fakhouri [10] proposed an identification algorithm for Wiener-Hammerstein model based on correlation analysis by applying the cross-correlation techniques to de-couple the identification of the linear dynamics from the characterization of the nonlinear element when the input is white Gaussian signal. Boutayeb and Darouach [18] proposed a method for recursive identification of MISO Wiener-Hammerstein model and by means of a transformation. They showed that parameters to be estimated were those of each subsystem of the initial and unique realization. Yoshine and Ishii [35] developed a new identification method for the G-model in the discrete time domain, based on the input-output causality and further developed a new formula for the estimation of nonlinear parameters in the G-model. Vandersteen and Shoukens [6] showed that it is possible to estimate the

nonparametric Frequency Response Functions (FRFs) of the linear dynamic elements of a Wiener- Hammerstein type of system. The identification is carried without measuring the signals over the static nonlinearity. An optimal two-stage identification Algorithm for the Hammerstein-Wiener Nonlinear system is given by Bai [1] where two static nonlinear elements surround a linear block. The Algorithm consists of two steps, the first is to find the Least Squares (LS) estimation of the product of parameters and the second is to extract the parameters of the system. Farooq in [2] used the recursive version of the Two-Stage Algorithm for developing different adaptive control schemes.

From the above, three basic parametric approaches can be distinguished for the identification of Hammerstein-Wiener models. The first one is the traditional iterative algorithm proposed by Narendra and Gallman [9]. The second approach is a correlation technique which was introduced by Billings [13] and Billings and Fakhouri [36], but it has restricted requirement on the input being white noise. The third approach is the most recent approach that has been introduced in Bai [1] which is based on least squares estimation (LSE) and singular value decomposition (SVD). Bai work is based on the work of Boutayeb et all [18], Chang and Luus [16] and Hsia [37] suggested for identification of Hammerstein models. Bai Algorithm is known as “Optimal Two Stage Identification Algorithm”

The concept of Genetic Algorithms (GAs) was developed by Holland [38], and subsequently in several research studies. An excellent discussion of GAs

and their implementations given by Goldberg [39] and Davis [19]. GAs have been successfully applied to engineering search and optimization problems such as system identification. Goldberg [39] has sited many examples in his book.

Kristinsson and Dumont [30] have shown that GAs can be used to solve the parameter estimation and control problem for a system described by the Auto-Regressive Moving Average (ARMA) model. It was shown that GAs can be applied effectively for identification on-line, as well as off-line, identification of both discrete and continuous systems. They were able to directly identify physical parameters or poles and zeros in both domains. Li [29] has demonstrated that GAs can be used to estimate a Hammerstein model. Also Yao and Sethares [40] used GAs to solve the parameter identification problem for linear and nonlinear digital filters. Zibo and Naghdy [23] applied GA techniques to identify parameters for an ARMAX model. Also they were able to apply it to SISO, MIMO and nonlinear system (Hammerstein type system)

CHAPTER 3

IDENTIFICATION OF HAMMERSTEIN-WIENER NON-LINEAR SYSTEMS USING GENETIC ALGORITHMS

In this chapter, Genetic Algorithm (GA) is used for the identification of the Hammerstein-Wiener non-linear systems. The equivalence between the Two Stage Identification Algorithm (TSIA) developed in the literature for the identification of these systems [4] is used to formulate the problem that can be solved using GA's. Some experiments are reported, and the performance of the GA based identification scheme is compared with that of the TSIA.

3.1 Introduction

The following nonlinear scalar stable discrete time dynamic system will be used throughout this chapter:

$$y(k) = \sum_{i=1}^{p} a_i \left\{ \sum_{l=1}^{q} d_l g_l[y(k-i)] \right\} + \sum_{j=1}^{n} b_j \left\{ \sum_{t=1}^{m} c_t f_t[u(k-j)] \right\} + \eta(k) \qquad (3.1)$$

Where $y(k), u(k), and\ \eta(k)$ are the system output, input and disturbance at time k respectively. This system model is shown in Figure 3.1.

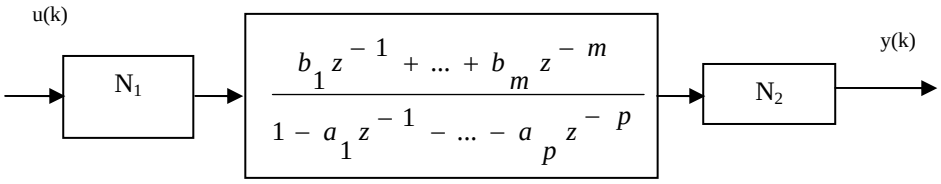

Figure 3.1: The non-linear system of equation (3.1)

The $g_i(.)'s\ and\ f_i(.)'s$ are nonlinear functions and $a = (a_1,.....,a_p)^{'}$, $b = (b_1,....,b_n)^{'}$, $c = (c_1,.....,c_m)^{'}$ and $d = (d_1,.....,d_q)^{'}$ denote the system parameter vectors. The above system is considered to be a special class of Hammerstein-Wiener nonlinear system [4].

The concept of Genetic Algorithms (GAs) was developed by Holland [38] in 1975, and subsequently in several research studies. Excellent discussions of GAs and their implementations are given by Goldberg [39] and Davis [19]. GAs have been successfully applied to engineering search and optimization problems such as system identification. Goldberg [39] has cited many examples in his book.

Kristinsson and Dumont [30] have shown that GAs can be used to solve the parameter estimation and control problem for a system described by the Auto-

Regressive Moving average (ARMA) model. It was shown that GAs can be applied effectively for both on-line and off-line identification of both discrete-time and continuous-time systems. They were able to directly identify the physical parameters or poles and zeros in both domains. Li [29] has demonstrated that GAs can be used to estimate a Hammerstein model. Also Yao and Sethares [40] used GAs to solve the parameter identification problem for linear and nonlinear digital filters. Zibo and Naghdy [23] applied GAs techniques to identify the parameters of an ARMAX model. Also they were able to apply it to SISO, MIMO and nonlinear system (Hammerstein type system) [23].

3.2 Brief description of Genetic Algorithms

GAs are stochastic parallel global search algorithms, which are based on the principle of natural genetic and natural selection. They operate on a population of current parameters (individuals) initially drawn at random and then an improvement on those current (best) population is sought. The current individual performance is assessed according to the objective function, which characterizes the problem to be solved. The objective function establishes the basis for selecting the best population. Then this population evolves from a generation to another generation through the application of genetic operators. A genetic algorithm in its simplest form uses three genetic operators: Reproduction, Crossover and Mutation.

Reproduction Stage: At the reproduction stage, a fitness value is derived from the raw individual performance measure given by the objective function, and used to bias the selection process. Highly fit individuals will have a higher probability of being selected to take part in the next stage than the less fit and, therefore, the average performance of this intermediate generation of individual is expected to increase.

Crossover and Mutation: After this stage, the selected individuals are modified through the application of the genetic operators (Crossover and Mutation), in order to obtain the next generation. The genetic operators manipulates the characters (genes) that constitute the chromosomes directly, following the assumption that certain genes code, on average, are fitter individual than others genes. Genetic operators can be divided into two main categories: **Crossover/Recombination:,** which causes pairs, or large groups, of individual to exchange genetic information with each other. **Mutation** causes individual genetic representations to be changed according to some probabilistic rule.

The last stage is that the resultant individual form both operators, crossover and mutation, will be evaluated and selected according to their fitness, and the process will continue.

Because GAs exploit strategies of genetic information and survival of the fittest to guide their search, they do not need to calculate the gradient or assume that the search space is differentiable or continuous. GAs simultaneously evaluate many points in the parameter space, so that they are more likely to

cover a global solution. GAs are very suitable for searching discrete, noisy, multimodel and complex space. Good references on the subject of GAs and their application can be found on references Goldberg [39], Davs [19], Coley [41], Gen and Cheng [42, 43] and Man, Tang and Kwang [44].

3.3 Bai's Identification algorithm

Bai [1] has shown that the following algorithm converges to the true parameters with no noise or white noise:

Bai's Uniqueness Assumption: Consider the system in equation (3.1):

- Assume that $ad^T \ and \ \ bc^T$ are not both zero.
- Assume that $\|a\|_2 = 1 \ \ and \ \ \|b\|_2 = 1$ and the signs of the first non-zero elements of $a's$ and $b's$ are positive.

Define the followings:

$$\theta(N) = (b_1c_1, \cdots, b_1c_m, \cdots, a_pd_1, \cdots, a_pd_q)^T$$
$$\theta(N) = (\theta_1, \cdots, \theta_{nm}, \theta_{nm+1}, \cdots, \theta_{nm+pq})^T$$

$$\Theta_{bc} = \begin{pmatrix} b_1c_1, & b_1c_2 & \cdots & ,b_1c_m \\ b_2c_1, & b_2c_2 & \cdots & ,b_2c_m \\ \vdots & \vdots & \ddots & \vdots \\ b_nc_1 & b_nc_2 & \cdots & b_nc_m \end{pmatrix}$$

$$\Theta_{ad} = \begin{pmatrix} a_1d_1, & a_1d_2 & \cdots & ,a_1d_q \\ a_2d_1, & a_2d_2 & \cdots & ,a_2d_q \\ \vdots & \vdots & \ddots & \vdots \\ a_pd_1 & a_pd_2 & \cdots & a_pd_q \end{pmatrix} \qquad (3.2)$$

$$\phi(k) = (f_1[u(k-1)],\cdots, f_m[u(k-1)],\cdots, f_1[u(k-n)],\cdots, g_1[u(k-p)],\cdots, g_q[u(k-p)])^T \qquad (3.3)$$

The system in equation (3.1) can now be written as

$$y(k) = \phi^T(k)\theta + \eta(k)$$

$$Y_N = (y(1), y(2),\cdots, y(N))^T \qquad ,\eta_N = (\eta(1),\cdots,\eta(N))^T$$

and

$$\Phi_N = (\phi^T(1),\cdots,\phi^T(N)) \qquad (3.4)$$

then

$$Y_N = \Phi_N\theta + \eta_N$$

Bai's Two Stage Identification Algorithm:

Considering the System in equation (3.1) under the uniqueness assumptions for a given set of data $u(k), y(k)$ *for* $k = 1,...,N.$

STEP 1: Calculate the least square estimates (LSE) of $\hat{\theta}(N)$

$\hat{\theta}(N) = \hat{\theta}_{ls}(N) = \left(\Phi_N^T \Phi_N\right)^{-1} \Phi_N^T Y_N$ from equations (3.3) and (3.4).

STEP 2: Construct $\hat{\Theta}_{bc}$ and $\hat{\Theta}_{ad}$ from $\hat{\theta}(N) = \hat{\theta}_{ls}(N)$ as in equation (3.2) and let

$$\hat{\Theta}_{bc}(N) = \sum_{i=1}^{\min(n,m)} \sigma_i \mu_i \nu_i^T, \quad \hat{\Theta}_{ad}(N) = \sum_{i=1}^{\min(p,q)} \delta_i \zeta_i \xi_i^T,$$

be their singular values decomposition (SVD) where

$\mu_i's (i = 1,2,...., p), \nu_i's (i = 1,2,..., m), \ \zeta_i's (i = 1,2,..., p)$ *and* $\xi_i's (i = 1,2,..., q)$

are n, m, p, q-dimensional orthonormal vectors respectively.

Where $\mu_i's$ & $\zeta_i's$ singular values, $\nu_i's$ & $\xi_i's$ right singular vectors, and $\sigma's$ & $\delta's$ left singular vectors.

Step 3: let s_μ denotes the sign of the first non-zero element of μ_1 and s_ζ denotes the sign of the first non-zero element of ζ_1. Define the estimate as follows:

$$\hat{b}(N) = s_\mu \mu_1, \ \hat{c}(N) = s_\mu \sigma_1 \nu_1, \ \hat{a}(N) = s_\zeta \zeta_1, \ \hat{d}(N) = s_\xi \delta_1 \xi_1$$

THEOREM 3.1 [1] : *Consider the system in equation (3.1) and the Two Stage Identification Algorithm under the Uniqueness Assumption. Then,*

1. *For any N > 0, if* ϕ_N *is full column rank and the disturbance* $\eta(N) \equiv 0$*, then*

$$\hat{a}(N) = a,\ \hat{b}(N) = b,\ \hat{c}(N) = c,\ \hat{d}(N) = d \qquad (3.5)$$

2. Let the disturbance $\eta(k)$ *be white with zero mean and finite variance and independent of* $u(k)$*. Suppose the input* $u(k)$ *is bounded and the regressor* $\phi(k)$ *is Persistently exciting (PE), i.e.,*

$$\alpha_2 I \geq \sum_{k=k_0}^{k_0+l_0} \phi(k)\phi^T(k) \geq \alpha_1 I > 0$$

For any $k_0 \geq 0$ *and some* $l_0 > 0$*. Then, with probability 1 and* $N \to \infty$*,*

$$\hat{a}(N) \to a,\ \hat{b}(N) \to b,\ \hat{c}(N) \to c,\ \hat{d}(N) \to d \qquad (3.6)$$

In [4] it was stated that the more interesting question is to find estimates $\hat{a}(N),\ \hat{b}(N),\ \hat{c}(N),\ \hat{d}(N)$ that minimizes

$$(\hat{a}(N),\hat{b}(N),\hat{c}(N),\hat{d}(N)) = \arg\min_{\bar{a},\bar{b},\bar{c},\bar{d}} \left\| Y_N - \hat{Y}_N(\bar{a},\bar{b},\bar{c},\bar{d}) \right\|^2_{A^T A} \qquad (3.7)$$

where

$$\hat{Y}(\hat{a},\hat{b},\hat{c},\hat{d}) = (\hat{y}(1,\hat{a},\hat{b},\hat{c},\hat{d}),\cdots,\hat{y}(N,\hat{a},\hat{b},\hat{c},\hat{d}))^T$$

with

$$\hat{y}(k,\hat{a},\hat{b},\hat{c},\hat{d}) = \sum_{i=1}^{p}\hat{a}_i\left\{\sum_{l=1}^{q}\hat{d}_l g_l\left[y(k-i)\right]\right\} + \sum_{j=1}^{n}\hat{b}_j\left\{\sum_{t=1}^{m}\hat{c}_t f_t\left[u(k-j)\right]\right\}$$

and; $\|X\|^2_{A^TA} = X^T A^T AX$, *and* A is some weighting matrix.

THEOREM 3.1 [1] : *Consider the system in equation (3.1) with some weighted matrix* A *as in equation (3.7) above. Let the least squares estimate in the first step of the Two Stage Identification algorithm be re-defined as:*

$$\hat{\theta}_{ls}(N) = (\overline{\Phi}_N^T\overline{\Phi}_N)^{-1}\overline{\Phi}_N^T\overline{Y}_N \quad \textit{with } \overline{\Phi}_N = A\Phi_N \quad \textit{and} \quad \overline{Y}_N = AY_N$$

Then, the estimates derived from the proposed Two Stage Identification Algorithm are the solution of

$$(\hat{a}(N),\hat{b}(N),\hat{c}(N),\hat{d}(N)) = \arg\min_{\hat{a},\hat{b},\hat{c},\hat{d}}\left\|Y_N - \hat{Y}_N(\hat{a},\hat{b},\hat{c},\hat{d})\right\|^2_{A^TA} \qquad (3.8)$$

For any A *such that all the singular values of* $A\Phi_N$ *are the same and non-zero*.

3.4 Proposed algorithm

The proposed algorithm consists of the following two phases:

1. Construct the matrix A, with identical singular values.
2. Apply GA based identification algorithm to minimize the objective function in equation (3.8). The GA for function Optimization in [45] is used as part of the GA based identification scheme.

Procedure for finding A: *The following algorithm is suggested to produce such matrix:*

$B = \Phi$
for $i = 1:10$

$B = 0.5(B + B(B^T B)^{-1})$

end
$A = B\ pinv(\Phi)$

By experiment, it was found that ten (10) iterations would produce the required A weighting matrix.

The above procedure produced the matrix A *such that all the singular values of* $A\Phi_N$ *are ones.*

3.5 Illustrative Examples

Example 3.1: The example used in Bai [1] will be used to illustrate the GA Parametric Identification algorithm results.

Consider the following Hammerstein-Wiener nonlinear system:

$$y(k) = a_1(d_1 y(k-1) + d_2 \sin(y(k-1)) + b_1(c_1 u(k-1) + c_2 u^2(k-1)) + b_2(c_1 u(k-2) + c_2 u^2(k-2)) + \eta(k)$$

where

$$a = (a_1) = 1,\ d = (d_1, d_2)^T = (0.5, 0.25)^T$$

$$b = (b_1, b_2)^T = (0.4472, -0.8944)^T,\quad c = (c_1, c_2)^T = (1,4)^T$$

for simulation, the following input will be used:

$$u(k) = 2\sin(2k) + 2\sin(4k) + 0.15\sin(6k) + 0.15\sin(8k) + 0.1\sin(10k)$$

and

$\eta(k)$ are i.i.d random variables uniformly in [-0.5,0.5]

We have run the GA based parameters identification algorithm and estimated parameters for various levels of noise (0.0, 0.25, 0.50, and 0.75) and we have the results are listed in table 3.1.

For N=100 data points with input u(k), the convergence is shown in the following Figure 3.2 to 3.8.

Noise Level	a1		b1		b2		c1	
	Bai	GA	**Bai**	GA	**Bai**	GA	**Bai**	GA
0.00	1.0000	1.0000	0.4472	0.4472	-0.8944	-0.8944	1.0000	0.9999
0.10	1.0000	1.0000	0.4472	0.4472	-0.8944	-0.8944	0.9998	0.9998
0.25	1.0000	1.0000	0.4472	0.4473	-0.8944	-0.8944	0.9993	1.0000
0.50	1.0000	1.0000	0.4472	0.4473	-0.8944	-0.8944	0.9997	0.9998
0.75	1.0000	1.0000	0.4473	0.4473	-0.8944	-0.8944	1.0005	1.0002
Noise Level	**c2**		**d1**		**d2**		**Error****	
	Bai	GA	**Bai**	GA	**Bai**	GA	**Bai**	GA
0.00	3.9999	3.9999	0.5000	0.5000	0.2500	0.2500	0.0000	0.0000
0.10	4.0000	3.9998	0.5000	0.4998	0.2512	0.2513	1.2640	1.4244
0.25	4.0002	4.0001	0.5000	0.5002	0.2526	0.2530	3.5020	3.8859
0.50	4.0003	4.0004	0.5000	0.5000	0.2570	0.2596	6.2100	6.8708
0.75	3.9993	3.9994	0.4998	0.5000	0.2530	0.2524	8.1431	9.0327

** The error between the Actual and the identified Models.

Table 3.1: Identified Model with Different Measurement Noise Level for both Bai and GA (system order $p=1, n=2, m=2, q=2$) for example 3.1.

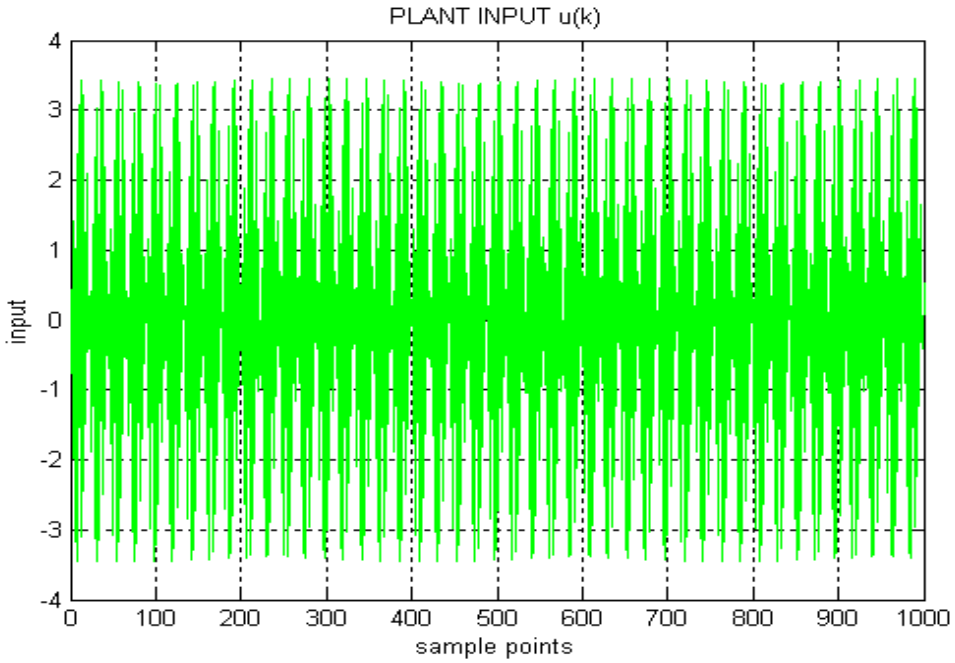

Figure 3.2: GA Based Identification Scheme for Bai's system: input u(k)

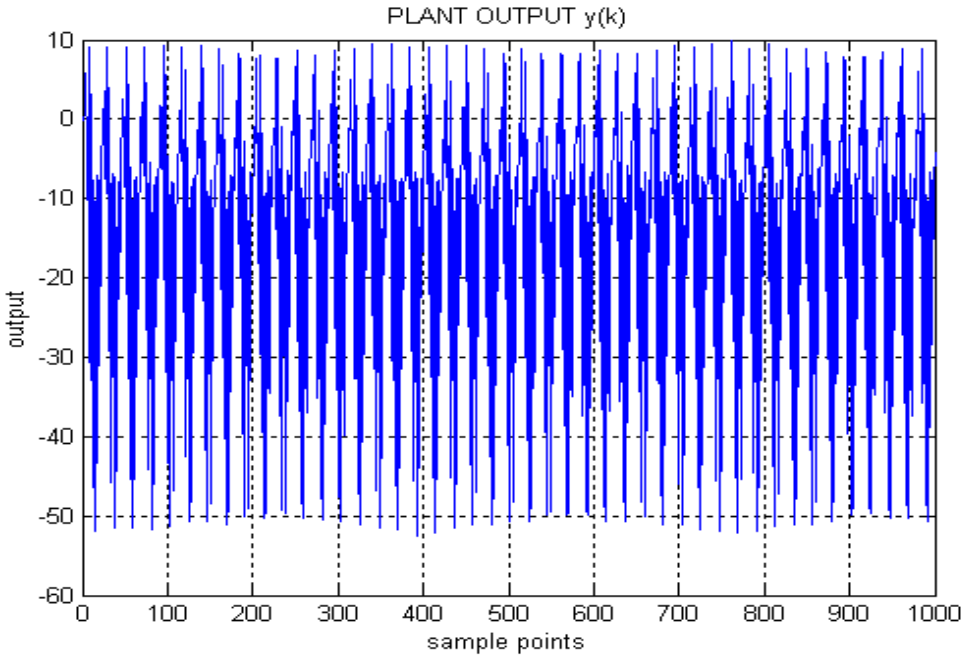

Figure 3.3: GA Based Identification Scheme for Bai's system: output y(k)

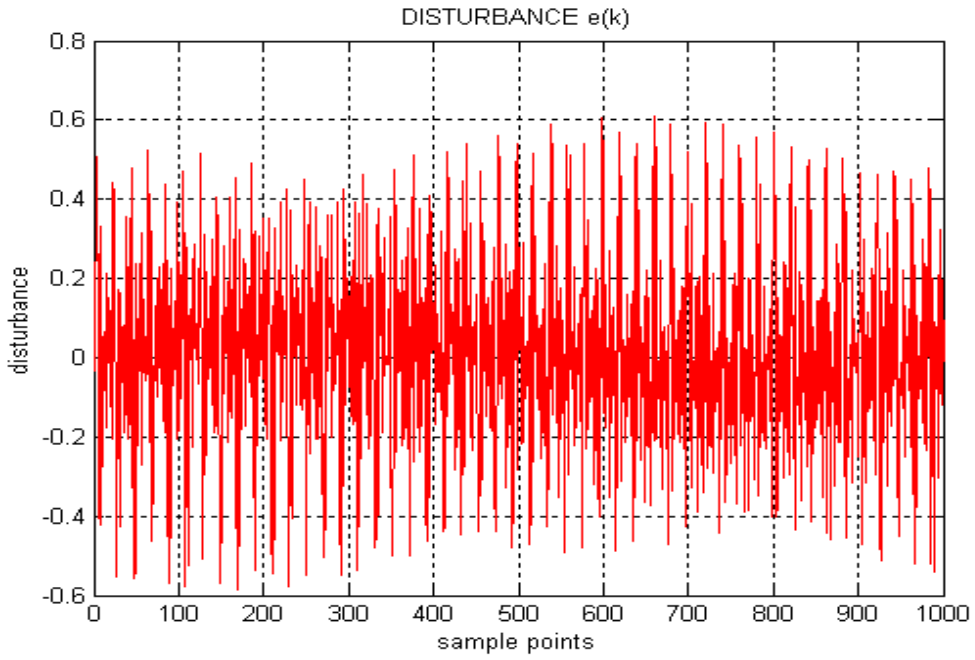

Figure 3.4: GA Based Identification Scheme for Bai's system: noise level[-0.75,0.75]

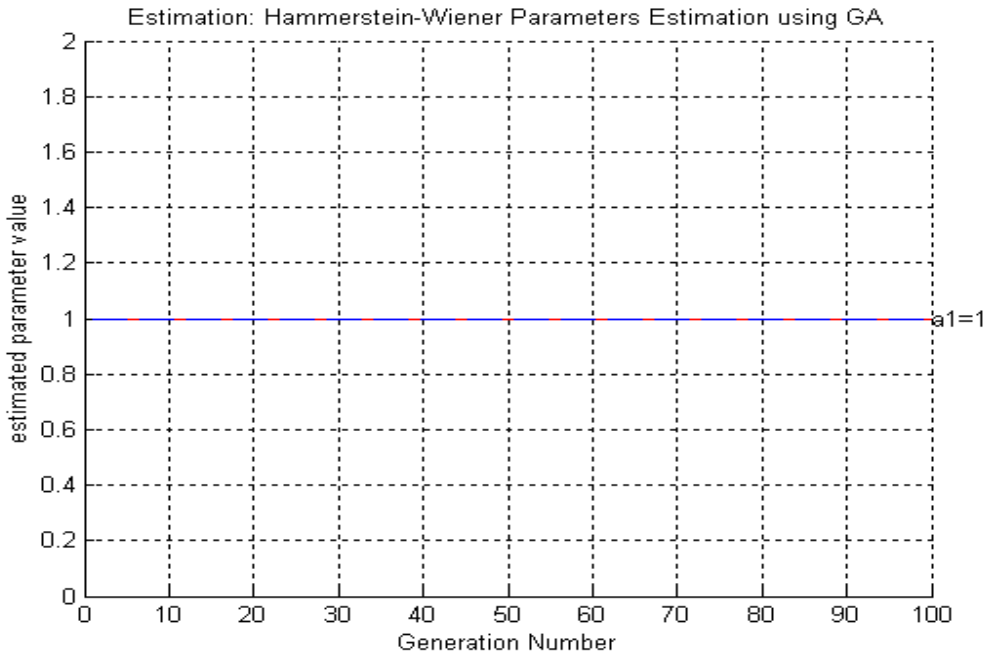

Figure 3.5: GA Based Identification Scheme for Bai's system for parameter "a_1"

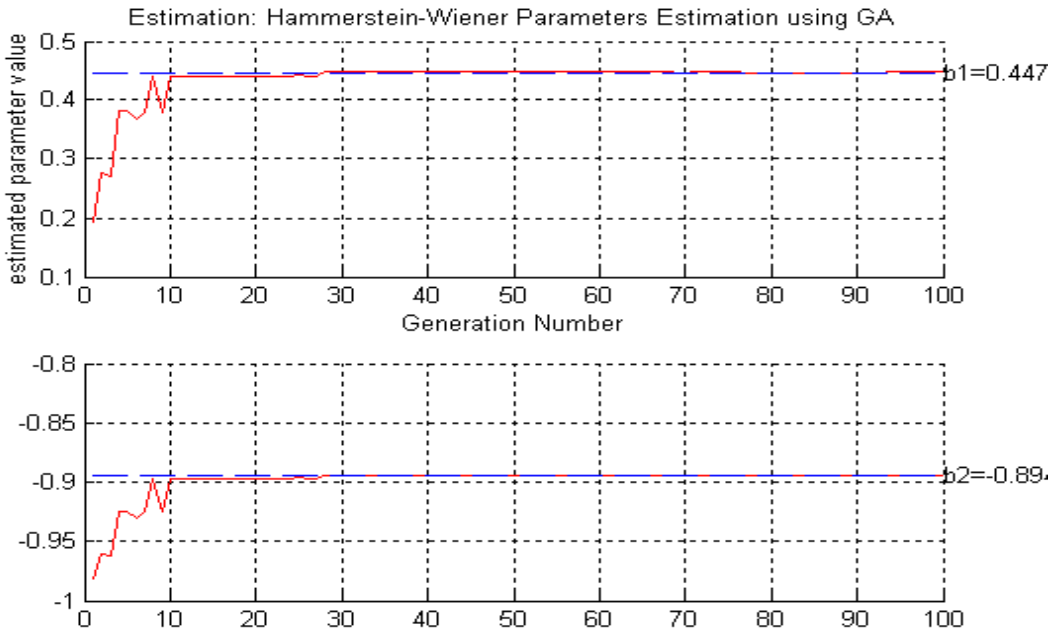

Figure 3.6: GA Based Identification Scheme for Bai's system for parameter "b_1&b_2"

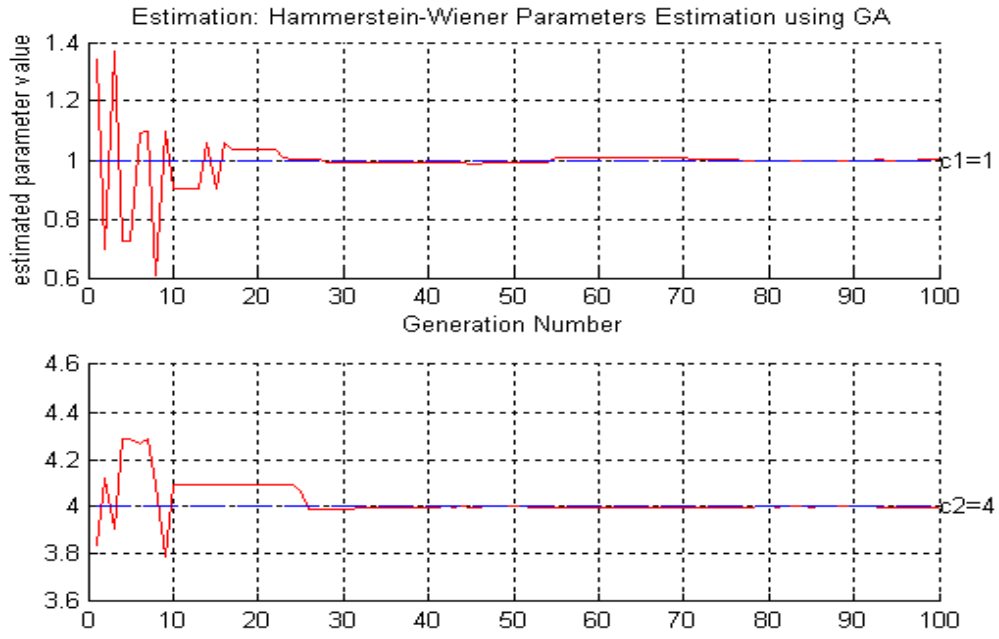

Figure 3.7: GA Based Identification Scheme for Bai's system for parameter "c_1 & c_2"

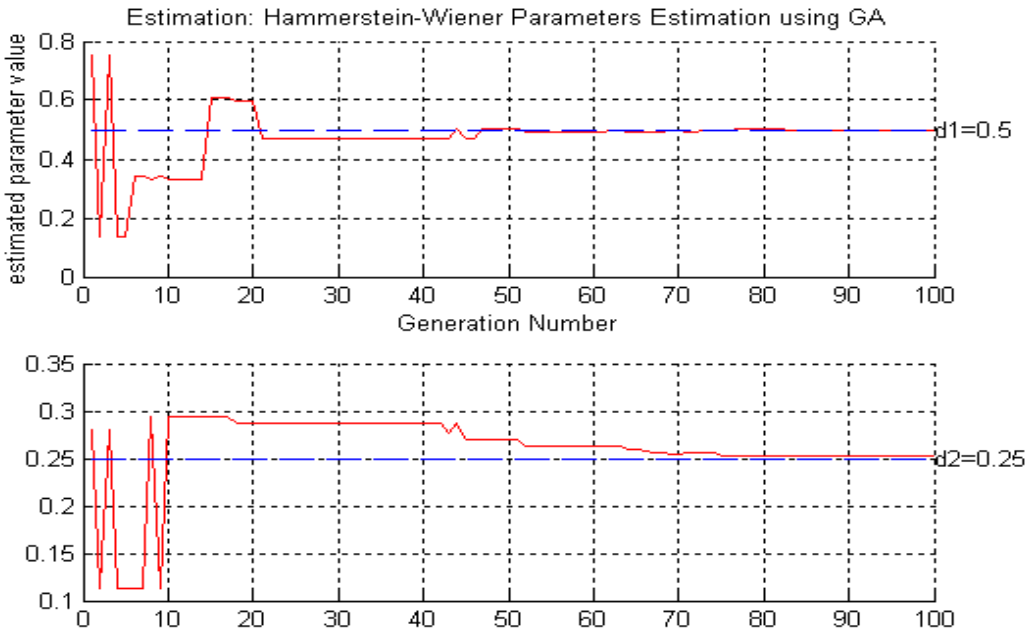

Figure 3.8: GA Based Identification Scheme for Bai's system for parameter "d_1 & d_2"

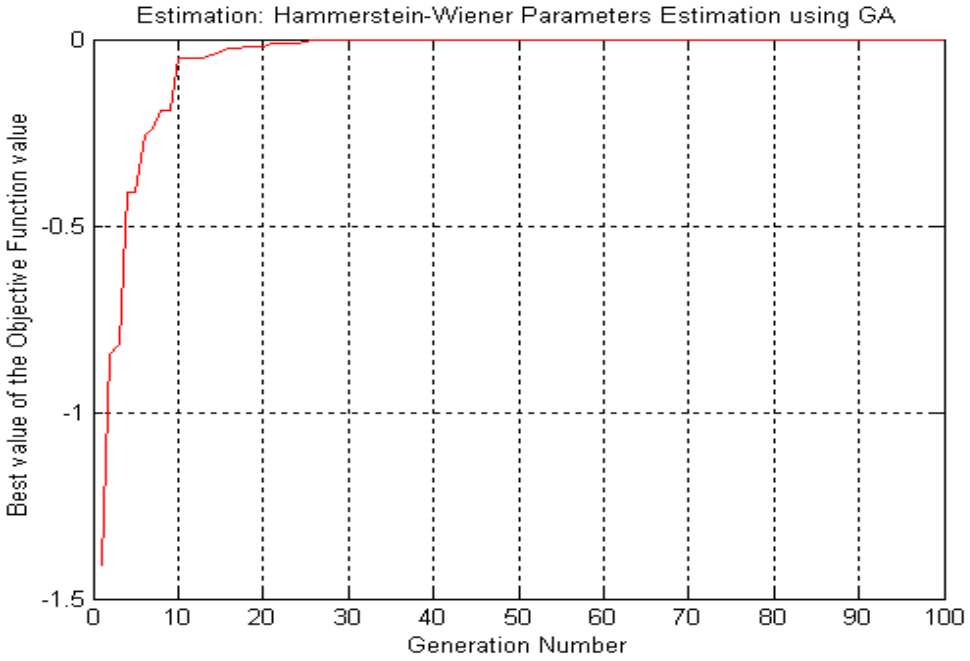

Figure 3.9: GA Based Identification Scheme for Bai's system error: true vs. estimated system

Example 3.2: This example has eight parameters to be identified:

Consider the following Hammerstein-Wiener nonlinear system:

$$y(k) = a_1(d_1 y(k-1) + d_2 \sin(y(k-1)) + a_2(d_1 y(k-2) + d_2 \sin(y(k-2)) + ...$$
$$b_1(c_1 u(k-1) + c_2 u^2(k-1)) + b_2(c_1 u(k-2) + c_2 u^2(k-2)) + \eta(k)$$

where

$$a = (a_1, a_2)^T = (0.6, 0.8)^T \quad , d = (d_1, d_2)^T = (0.5, 0.15)^T$$
$$b = (b_1, b_2)^T = (0.4472, 0.8944)^T, \quad c = (c_1, c_2)^T = (1, 0.1)^T$$

for simulation, the following input will be used:

$$u(k) = 2\sin(2k) + 2\sin(4k) + 0.15\sin(6k) + 0.15\sin(8k) + 0.1\sin(10k) +$$
$$0.1\sin(12k) + 0.1\sin(14k)$$

and

$\eta(k)$ are i.i.d random variables uniformly in [-0.75,0.75]

We have ran the GA based parameters identification algorithm and estimated parameters for various levels of noise (0.0, 0.25, 0.50, and 0.75) and we have the following results:

Noise Level	a1		a2		b1		b2	
	Bai	GA	**Bai**	GA	**Bai**	GA	**Bai**	GA
0.00	0.6000	0.6002	0.8000	0.7998	0.4472	0.4473	0.8944	0.8944
0.10	0.6046	0.6054	0.7965	0.7959	0.4505	0.4506	0.8928	0.8927
0.25	0.6250	0.6247	0.7806	0.7809	0.4481	0.4481	0.8940	0.8940
0.50	0.6060	0.6028	0.7954	0.7979	0.4446	0.4444	0.8957	0.8958
0.75	0.5688	0.5696	0.8225	0.8219	0.4545	0.4543	0.8907	0.8908
Noise Level	**c1**		**c2**		**d1**		**d2**	
	Bai	GA	**Bai**	GA	**Bai**	GA	**Bai**	GA
0.00	1.0000	1.0000	0.1000	0.1002	0.5000	0.5001	0.1500	0.1498
0.10	1.0018	1.0018	0.0995	0.0995	0.5002	0.5004	0.1512	0.1515
0.25	0.9950	0.9950	0.1005	0.1006	0.4967	0.4966	0.1374	0.1375
0.50	0.9804	0.9804	0.1054	0.1072	0.4877	0.4869	0.1213	0.1218
0.75	1.0419	1.0421	0.0928	0.0925	0.4956	0.4954	0.0253	0.0271

Table 3.2: Continuation- Identified Model with Different Measurement Noise Level for both Bai and GA (system order $p = 2, n = 2, m = 2, q = 2$) for example 3.2.

Noise Level	0.0	0.1	0.25	0.50	0.75
Bai	0.00	0.4530	1.0552	2.1021	3.2204
GA	0.0652	0.4811	1.1510	2.2443	3.9559

Table 3.3: The error between the Actual and the identified Models for both schemes (Bai and GA).

For N=100 data points with input u(k), the convergence is shown in the following Figure 3.9 to 3.14.

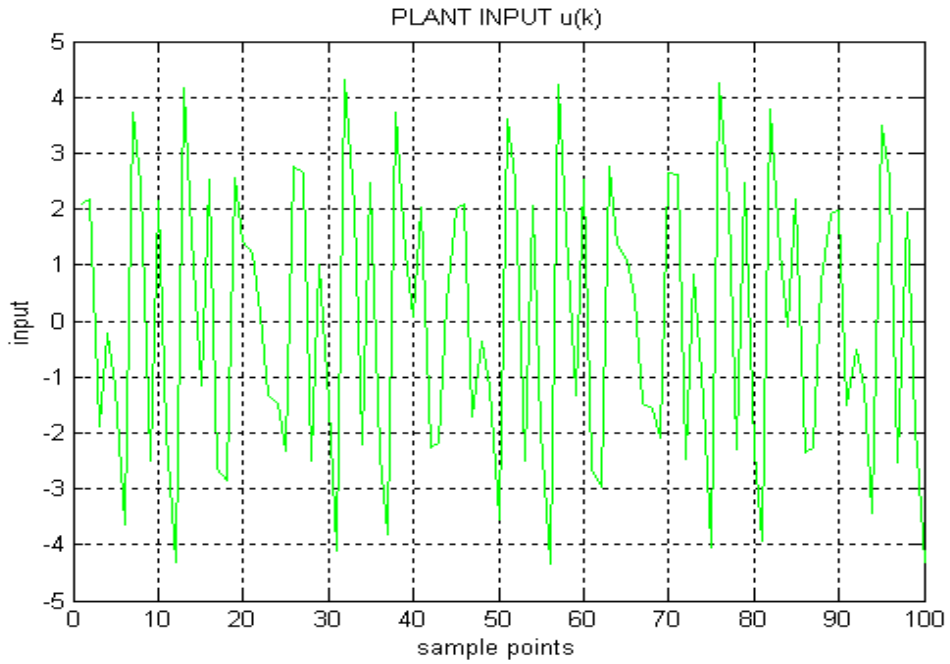

Figure 3.10: GA Based Identification Scheme for Bai's system: input u(k)

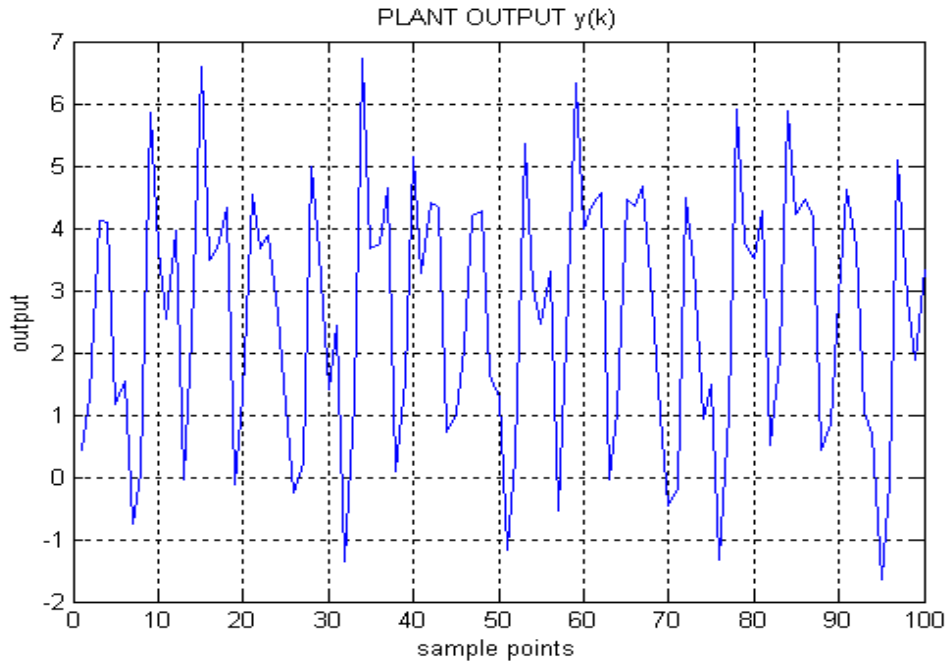

Figure 3.11: GA Based Identification Scheme for Bai's system: output y(k)

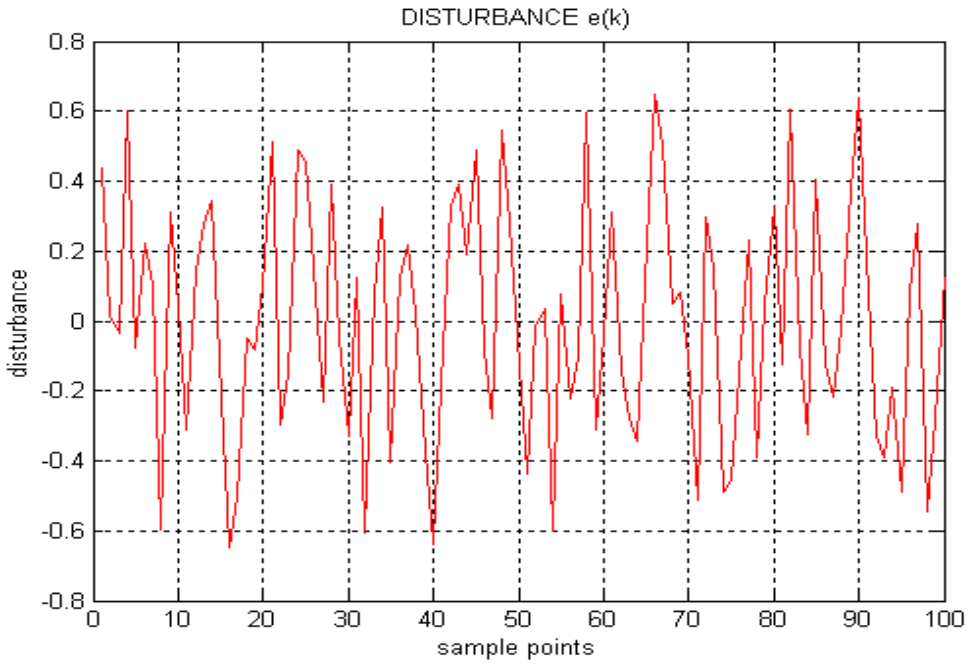

Figure 3.12: GA Based Identification Scheme for Bai's system: noise level [-0.75,0.75]

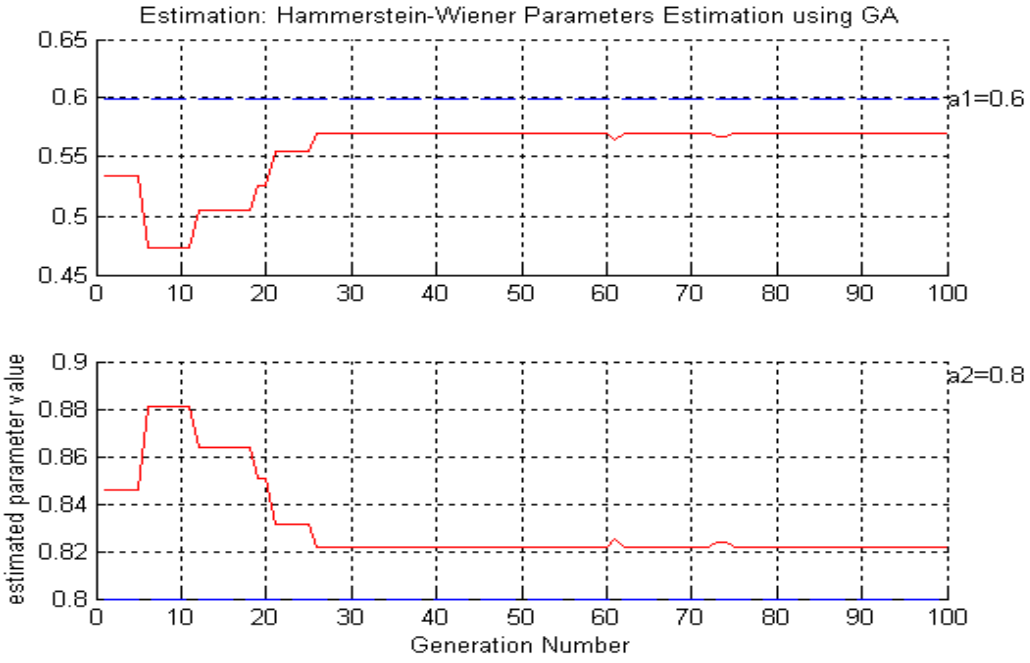

Figure 3.13: GA Based Identification Scheme for Bai's system for parameter "a_1 & a_2"

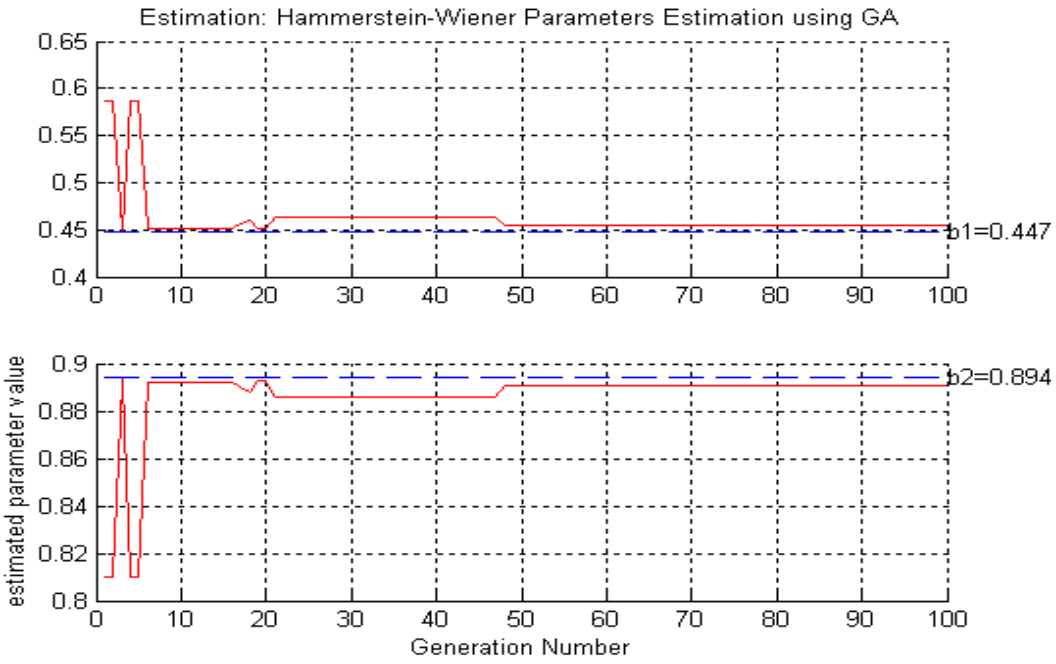

Figure 3.14: GA Based Identification Scheme for Bai's system for parameter "b_1 &b_2"

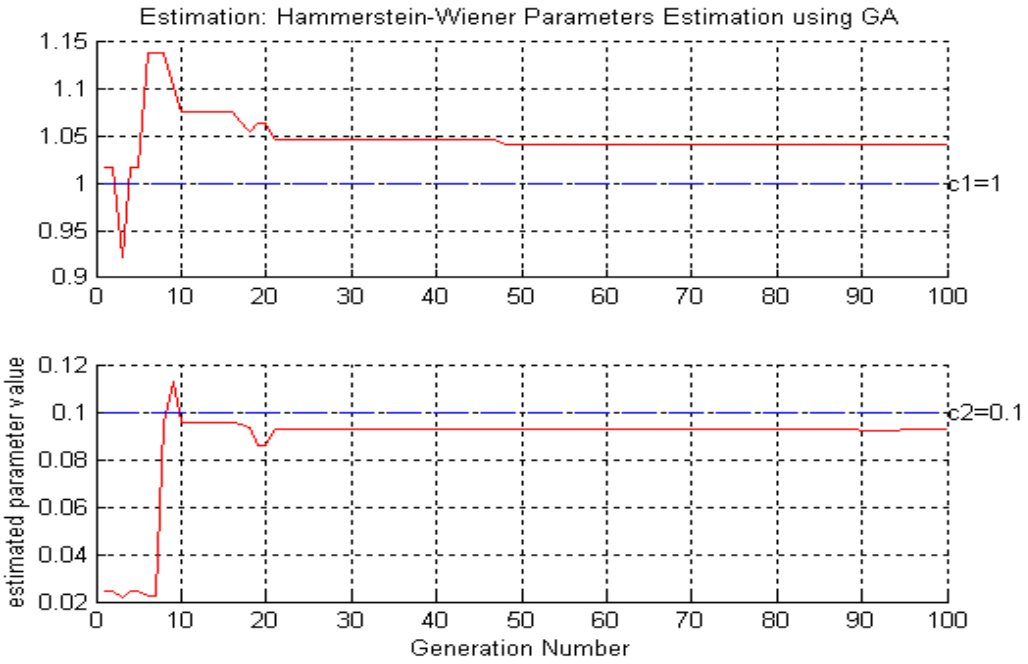

Figure 3.15: GA Based Identification Scheme for Bai's system for parameter "c_1 & c_2"

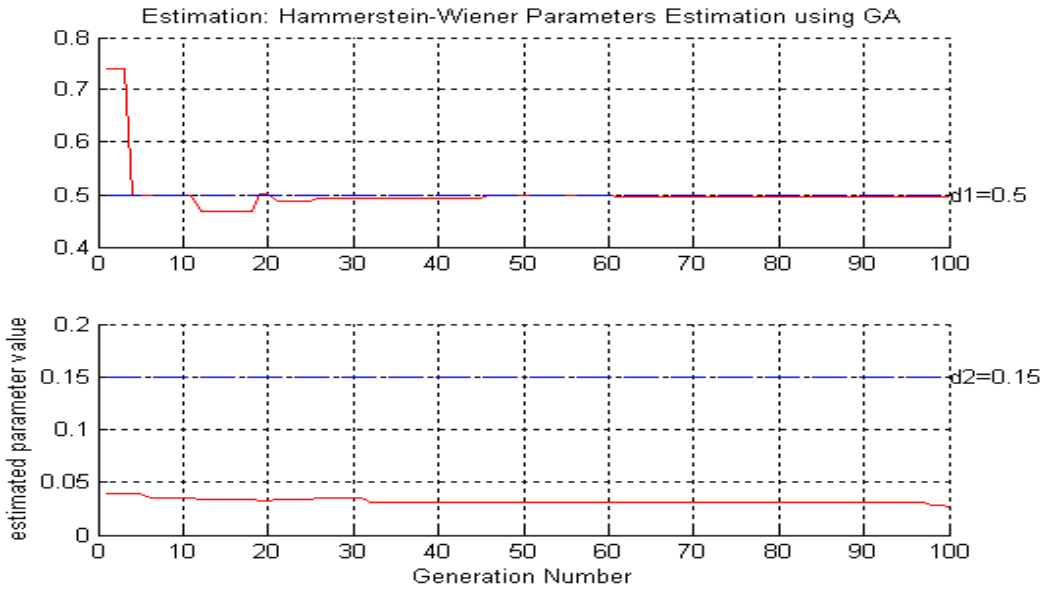

Figure 3.16: GA Based Identification Scheme for Bai's system for parameter "d_1 & d_2"

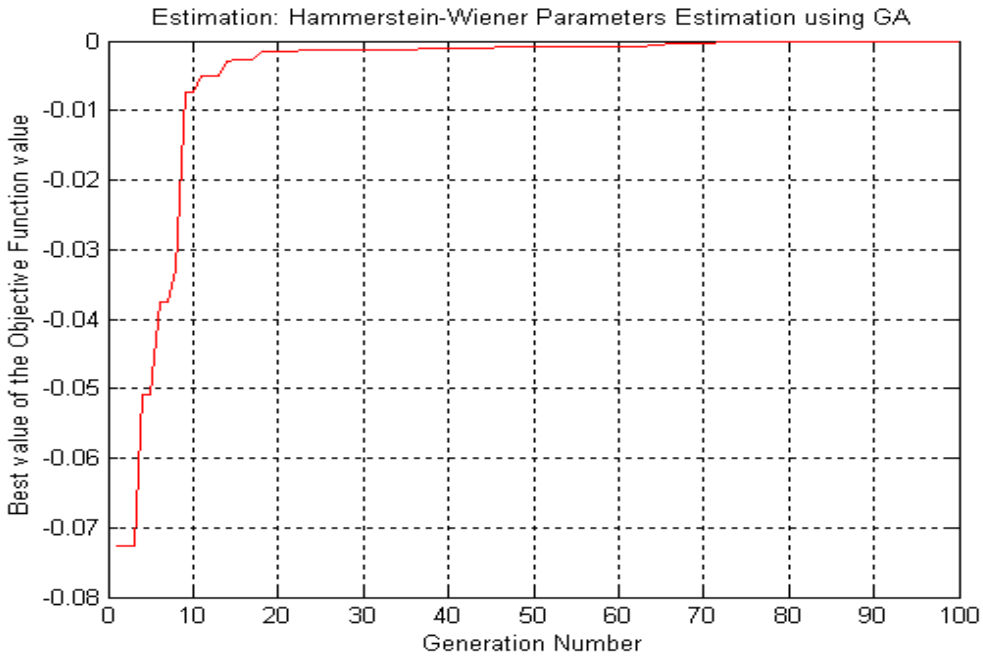

Figure 3.17: GA Based Identification Scheme for Bai's system error: true vs. estimated system

It can be seen from the above two examples (tables and figures) that the result of the GA based identifications scheme has produced very comparable results with the estimated parameters very close to the true values of the original system parameters. We also noticed in both examples that d2 is not converging as good as the other estimated parameters.

The examples were also used to compare the performance of the Two Stage methodology and the GA based identifications scheme to identify the parameters of the Hammerstein-Wiener nonlinear system. The table lists the objective functions (error between the plant and the identified model) of both the Two-Stage, and the proposed algorithms for Example 3.1. As expected, it shows that as the noise level increases, the error between the true system and the identified model increases.

3.6 Concluding Remarks

In this chapter, a new algorithm for identification of Bai model is proposed. The two stage algorithm of Bai [1] was formulated in such a way so that GA can be used to solve it. The proposed GA based identification scheme has produced very encouraging results with the estimated parameters very close to the true values of the original system parameters. It also produced results that are very close to the results produced by Bai's Two Stage identification algorithm.

CHAPTER 4

RECURSIVE IDENTIFICATION OF HAMMERSTEIN MODEL BY MINIMIZING THE H_∞ NORM OF THE MISMATCH ERROR (MODIFIED APPROACH)

In this chapter, we consider identification of a special class of nonlinear systems using Hammerstein models. Hammerstein models are special in the sense that they can be transformed into linear models and linear systems techniques may be applicable. A modified iterative identification algorithm is proposed to obtain a model that minimizes the H_∞ norm of the mismatch error. This algorithm is based on the work done by Al-Amer and Al-Sunni [11, 20]. Illustrative examples are given with concluding remarks.

4.1 Introduction

Hammerstein models have been successively used in modeling some physical systems [5]. As we have discussed in Section 2.2, Hammerstein model

consists of a memoryless nonlinear subsystem $N(.)$ followed by a linear shift invariant subsystem $G(z)$ as shown in Figure 2.1. The intermediate variable $x(k)$ is not measurable.

Several algorithms have been used to identify Hammerstein models that have been proposed. They can be classified into two groups [11, 12]. In the first class, iterative identification of the linear and nonlinear parts are done [1, 13]. A major problem in these algorithms is assuring the convergence of the iterations [36]. The second group includes noniterative algorithms that simultaneously identify $N(.)$ and $G(z)$ [8]. In general, these are expected to lead to more accurate models but they have a larger number of parameters to be estimated.

The least squares and mean square error techniques are often used to obtain the models. An algorithm to minimize the H_∞ gain is presented in [46]. In this chapter, we proposed a modified identification algorithm to obtain a Hammerstein model that minimizes the H_∞ norm of the mismatch error based on the work done by Al-Amer and Al-Sunni [11].

4.2 Problem Statement

Given a set of input-output data $\{x(k), y(k)\}$ and two integers $m \ and \ n$, it is required to fit the data to a Hammerstein model. The system to be identified is a nonlinear single-input-single-output (SISO) system. The nonlinear block is assumed to be of the form

$$u(k) = \sum_{i=1}^{m} v_i x^i(k) \qquad (4.1)$$

and the linear block is assumed to be a stable shift invariant system described by

$$\mathrm{y(k)} = \frac{\sum_{i=0}^{n} b_i\, q^{-i}}{1 + \sum_{i=1}^{n} a_i\, q^{-i}}\; u(k) + \mathrm{e(k)}. \qquad (4.2)$$

The sensor noise $e(k)$ is assumed to bounded.

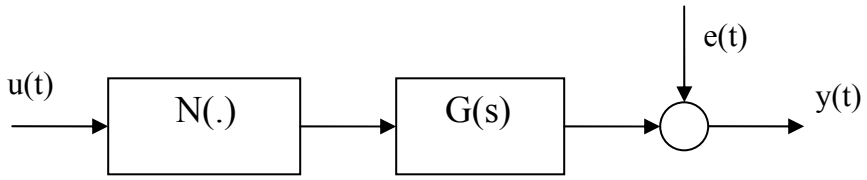

Figure 4.1: Hammerstein Model Under Investigation

The linear block

$$G(q^{-1}) = \frac{\sum_{i=0}^{n} b_i\, q^{-i}}{1 + \sum_{i=1}^{n} a_i\, q^{-i}}$$

is assumed to be a stable shift invariant system.

Let the model parameters be denoted by

$$\theta_{\mathrm{v}} = [v_1 v_2 \cdots v_m]^T,$$
$$\theta_{\mathrm{a}} = [a_1 a_2 \cdots a_n]^T,$$
$$\theta_{\mathrm{b}} = [b_0 b_1 \cdots b_n]^T$$

Note that the parameters of the model can not be uniquely determined and the following constraint is introduced.

$$v_1 = 1.$$

The other coefficients are normalized accordingly. By doing the following artificial inputs [30]

$$z_{\mathrm{i}}(k) = x^i(k)$$

The Hammerstein model can be viewed as a linear m-input 1-output system (see Figure 4.2 for illustration).

Let $Z_{\mathrm{i}}(z), U(z), Y(z)$ and $E(z)$ denote the Z-transform of $z_{\mathrm{i}}(k), u(k), y(k)$ and $e(k)$ respectively then

$$Y(z)=\frac{\sum_{i=0}^{n} b_i z^{-i}}{1+\sum_{i=1}^{n} a_i z^{-i}} \sum_{i=0}^{m} v_i Z_i(z)+E(z) \qquad (4.3)$$

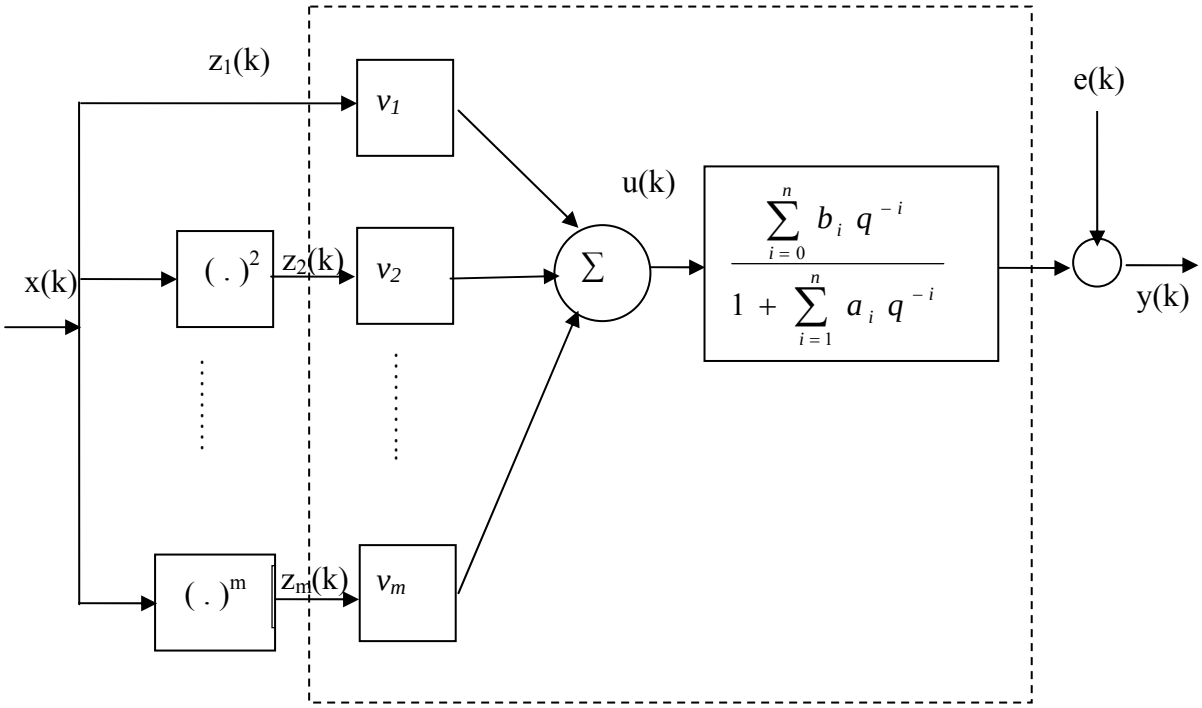

Figure 4.2: An Alternative Representation of Hammerstein Model

Next, we will present an identification algorithm to obtain v_i, a_i, and b_i so that the H_∞ norm of the mismatch error is minimized. More precisely the following problem is to be solved.

$$\min_{a_i,b_i,v_i} \left\| Y(z) - \frac{\sum_{i=0}^{n} b_i z^{-i}}{1+\sum_{i=1}^{n} a_i z^{-i}} \sum_{i=0}^{m} v_i Z_i(z) \right\|_{\infty} \qquad (4.4)$$

According to Lawson generalized Algorithm [47], the solution for the H_{∞} can be found via Weighted H_2 optimization problem as follows:

$$\| G_1 - G_2 \|_{\infty} = \| (G_1 - G_2) W \|_2$$

Where W is the weight matrix

That is H_{∞} approximation problem is obtained by a weighted least squares problem in the frequency domain.

4.3 The Proposed Identification Algorithm

In this section we present a modified procedure to identify a Hammerstein model from a set of input output data to solve the optimization problem in equation (4.4). The proposed algorithm is a modification and an extension of the algorithm developed in [11]. The modification was based on the work done in

[20]. The solution to the H_∞ approximation problem is obtained by solving a sequence of weighted least squares problems. Each iteration of the algorithm involves two approximation problems. In the first, θ_b is assumed to be available and a least squares problem is solved for θ_v and θ_a. In the second θ_v is fixed to its latest value and a least squares problem is solved for θ_a and θ_b. To insure the uniqueness of the parameter set, a scaling is done to force $v_1 = 1$ and the other parameters are adjusted accordingly.

4.4 Implementation

The algorithm uses samples of the Z-transform of the input and output on the unit circle. This is obtained by computing the N-point discrete Fourier transform of $y(k)$ and $Z_i(k)$. The computed Fourier transforms denoted by $Y\left(e^{j\omega_k}\right)$ and $Z_i\left(e^{j\omega_k}\right)$ are basically samples of $y(k)$ and $Z_i(k)$ on the unit circle at the frequencies $\omega_k = \frac{2\pi k}{N}$.

Define

$$\phi_v(k) = \left[Z_1\left(e^{j\omega_k}\right) Z_2\left(e^{j\omega_k}\right) \cdots Z_m\left(e^{j\omega_k}\right) \right]$$

$$\phi_a(k) = \left[Y\left(e^{j\omega_k}\right)e^{-j\omega_k}\, Y\left(e^{j\omega_k}\right)e^{-2j\omega_k} \cdots Y\left(e^{j\omega_k}\right)e^{-nj\omega_k} \right]$$

$$\phi_b(k) = \left[1 \;\; e^{-j\omega_k} \cdots e^{-nj\omega_k} \right]$$

$$\bar{\phi}_b(k) = \phi_v(k)\theta_v \quad \phi_b(k)$$

and

$$\Phi_v = \begin{bmatrix} \phi_v(1) \\ \phi_v(2) \\ \vdots \\ \phi_v(N) \end{bmatrix}, \quad \Phi_b = \begin{bmatrix} \phi_b(1) \\ \phi_b(2) \\ \vdots \\ \phi_b(N) \end{bmatrix}, \quad \Phi_a = \begin{bmatrix} \phi_a(1) \\ \phi_a(2) \\ \vdots \\ \phi_a(N) \end{bmatrix}$$

Each iteration of the proposed algorithm involves solving two weighted least squares approximation problems. In the first problem, θ_b is fixed to its latest computed value and the following weighted least square problem is solved for θ_v and θ_a.

$$\min_{\theta_a, \theta_v} \left\| \left(Y(e^{j\omega_k}) + \phi_a(k)\,\theta_a - \phi_b(k)\,\theta_b\,\phi_v(k)\,\theta_v \right) W\left(e^{j\omega_k}\right) \right\|_2 \qquad (4.5)$$

Where $W\left(e^{j\omega_k}\right)$ is the weighting matrix.

To insure the uniqueness of the parameter set, a scaling is done to force v_1 to be one and the other parameters are adjusted accordingly. In the second problem, θ_v is fixed at its latest value and the following problem is solved for θ_a and θ_b.

$$\min_{\theta_a,\theta_b}\left\| \left(Y(e^{j\omega_k}) + \phi_a(k)\ \theta_a - \phi_b(k)\ \theta_b\ \phi_v(k)\ \theta_v\right) W\left(e^{j\omega_k}\right)\right\|_2 \qquad (4.6)$$

An initial guess for the linear part in the first problem is to assume that the linear block is an all-pole system (i.e., $b_0 = 1$, $b_i = 0$ for $i \neq 0$). The frequency weighting at each iteration is computed as the product of the previous weight and the error corresponding to the latest available model. An initial value of the weight is given by $w^1(e^{j\omega_k}) = 1\ \forall k$. w is a column vector of the weighting matrix W. A summary of the proposed algorithm is given below.

4.5 The identification Algorithm

Given m, n, N and $\{(x(k), y(k), k = 1,2,...)\}$

Step 0: Compute the N-point FFT of $y(k)$ and $z_i(k)$.

Step 1: Let $\nu_0 = 1, \nu_i = 0,\ for\ i \neq 0\ \ l = 1, w_k^l = 1$ for $k \in [1, N]$.

Step 2: Compute θ_v and θ_a using

$$\begin{bmatrix}\theta_a \\ \theta_v\end{bmatrix} = \left(\Phi_1{}^T\ W\ \Phi_1\right)^{-1} \Phi_1\ W\ Y$$

where $\Phi_1 = \left[\Phi_a\ \ \Phi_v\right]$ and $\mathrm{W} = \mathrm{diag}(\ w^1(1), \mathrm{w}^1(2), \cdots, \mathrm{w}^1(\mathrm{N})\ )$

Step 3: Scale θ_b and θ_v using $\theta_b = v_1\ \theta_b, \quad \theta_v = \dfrac{\theta_v}{v_1}$

Step 4: Update the weight using

$$W^{l+1}(k) = \frac{W^l(k)}{\alpha}\left|Y(k) - \frac{\phi_b(k)\theta_b}{1+\phi_a(k)\theta_a}\phi_v(k)\theta_v\right|$$

where α *is selected such that* $\left\|W^{l+1}\right\|_2 = 1$ *or* $\max_k\left|W^{l+1}\right| = 1$

Step 5: Now fix θ_v and Compute θ_a and θ_b by solving another least squares problem

$$\begin{bmatrix}\theta_a \\ \theta_b\end{bmatrix} = \left(\Phi_2{}^T W \Phi_2\right)^{-1}\Phi_2{}^T W\ Y$$

where

$$\Phi_2 = \begin{bmatrix}\Phi_a & \Phi_b\end{bmatrix}$$

Step 6: Update the weight as in step 4.

Step 7: Set $l = l+1$ and go to step 2.

Remark:

The algorithm is terminated after a fixed number of iteration and the identified model is selected as the one that gives the least error.

4.6 Illustrative Examples

Two examples are presented here to illustrate the algorithm.

4.6.1 Example 1

The nonlinear system being identified is shown in Figure 4.3. The apriori information $n = 4, m = 4$ are assumed. The input x(k) is generated as a uniformly distributed sequence of magnitude 1. The measurement noise is uniformly distributed with different levels (0.0,0.02 and 0.05). The algorithm uses 1024-point FFT for computing the identified model. The identified model (after 5 iterations) is given in Table 4.1.

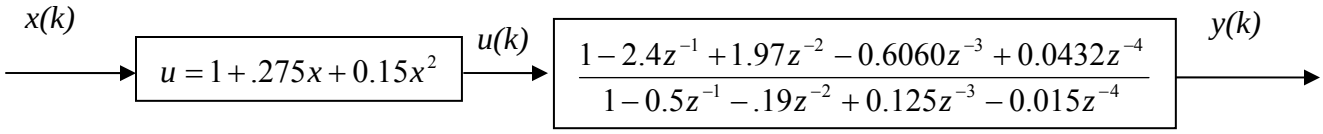

Figure 4.3: True Model, Example 1

Noise	Nonlinear Block	Linear Block	$\|Error\|_\infty$
0	$u = 1 + .275x + 0.15x^2$	$\frac{1 - 2.4z^{-1} + 1.97z^{-2} - 0.6060z^{-3} + 0.0432z^{-4}}{1 - 0.5z^{-1} - .19z^{-2} + 0.125z^{-3} - 0.015z^{-4}}$	0.0
0.01	$u = 1 + .275x + 0.15x^2$	$\frac{1 - 2.4048z^{-1} + 1.9797z^{-2} - 0.6122z^{-3} + 0.0444z^{-4}}{1 - 0.5048z^{-1} - .1893z^{-2} + 0.1259z^{-3} - 0.0153z^{-4}}$	0.0372
0.05	$u = 1 + .275x + 0.15x^2$	$\frac{1 - 2.4382z^{-1} + 2.0632z^{-2} - 0.683z^{-3} + 0.0651z^{-4}}{1 - 0.5382z^{-1} - .1694z^{-2} + 0.1335z^{-3} - 0.0206z^{-4}}$	0.1862

$\|Error\|_\infty$ is calculated using MATLAB function: norm(Error,inf)

Table 4.1: Identified Model with Different Measurement Nose Level for Example 1

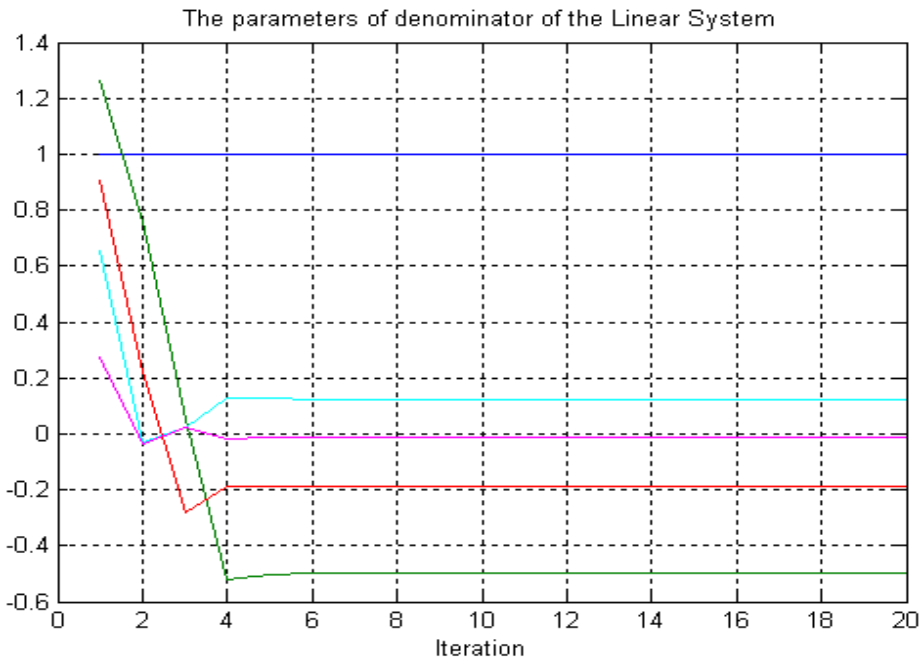

Figure 4.4: Parameters Convergence: 4th order model- a's coefficient (Ex. 1)

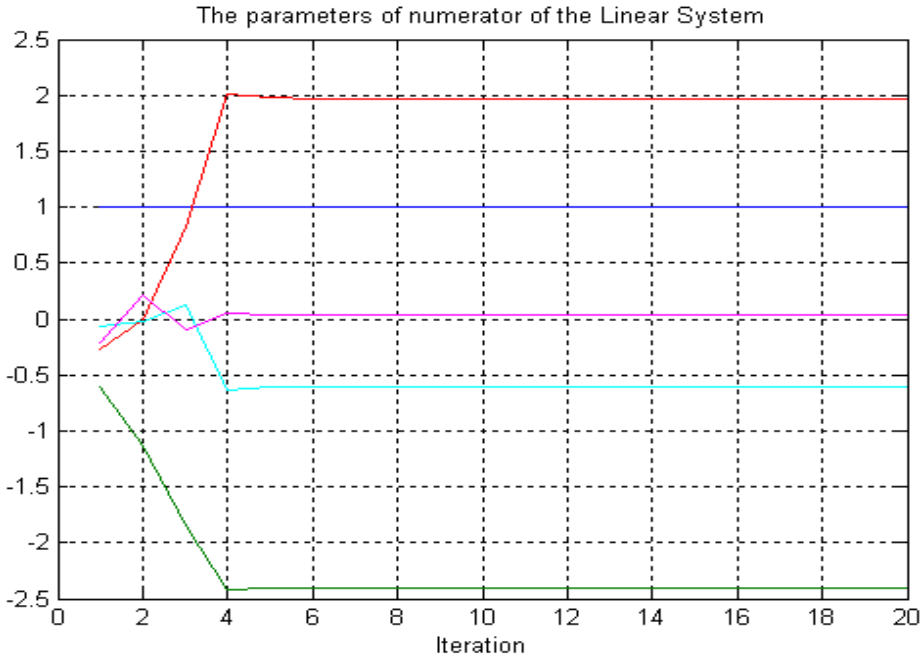

Figure 4.5: Parameters Convergence: 4th order model- b's coefficient (Ex. 1)

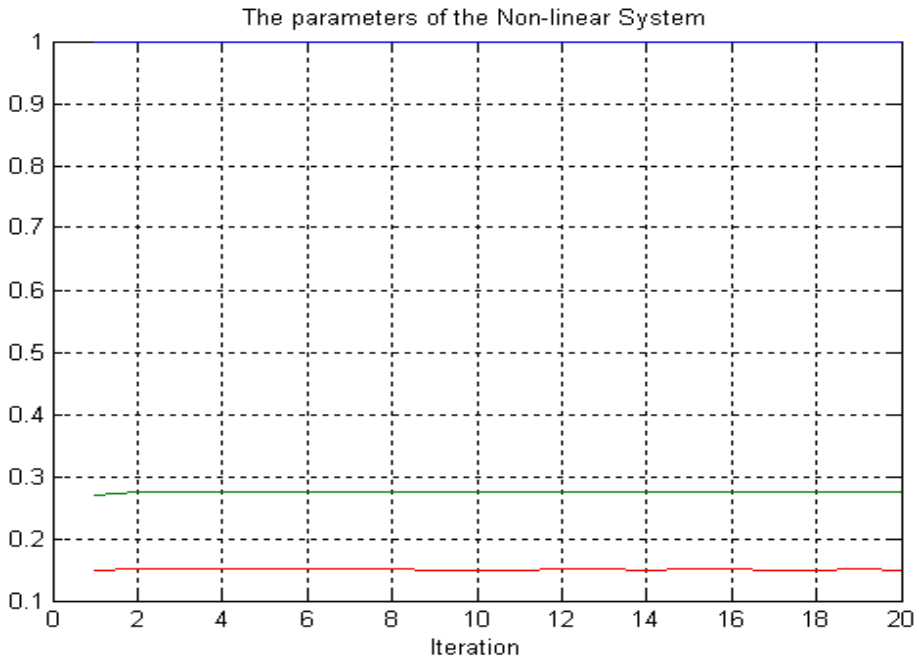

Figure 4.6: Parameters Convergence: the nonlinear block (Example 1)

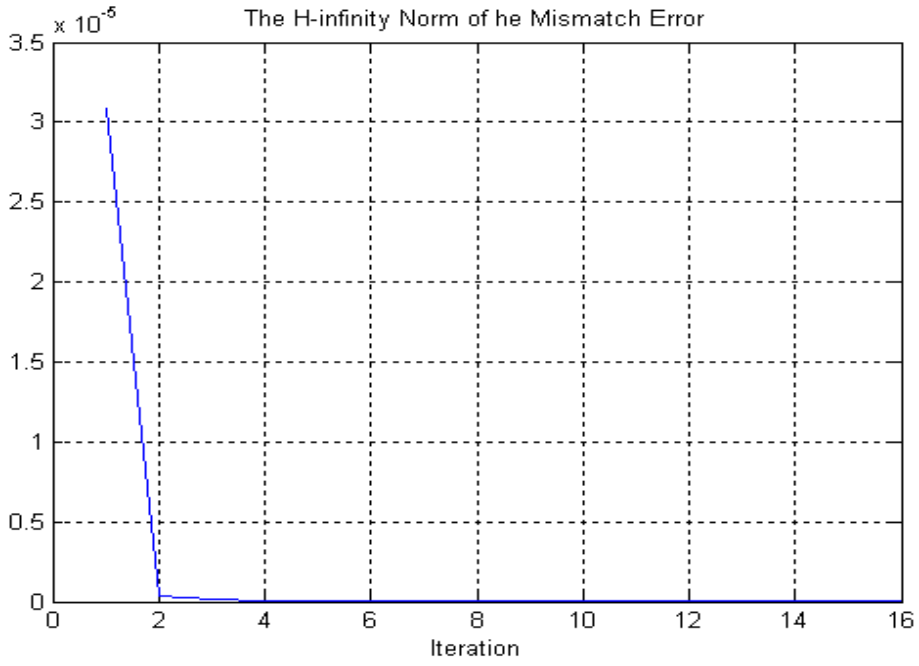

Figure 4.7: The H_∞ Norm of the mismatch error (Example 1)

In this example and analyzing the trajectory of the estimated parameters, it is observed that the results obtained, as in Table 4.1 and Figures 4.4 to 4. 7, are very comparable to the true plant parameters. Also, it is observed that the algorithm converged to the best estimates at the 4^{th} iteration.

4.6.2 Example 2

The nonlinear system being identified is shown in Figure 4.8. The apriori information $n = 3$, $m = 3$ are assumed. The input x(k) is generated as a uniformly distributed sequence of magnitude 1. The measurement noise is uniformly distributed with different levels (0.0, 0.02 and 0.05). The algorithm uses 1024-point FFT for computing the identified model. The identified model (after 5 iterations) is given in Table 4.2.

4.6.3 Example 3

The nonlinear system being identified is shown in Figure 4.9. The apriori information $n = 3$, $m = 3$ are assumed. However, for example, the poles were chosen to very close to the boundary of the unit cycle (-0.55+0.83i;-0.55-0.83i;-0.893). The objective of this example is to test the algorithm performance when such condition exists. The input x(k) is generated as a uniformly distributed sequence of magnitude 1. The measurement noise is uniformly distributed with different levels (0.0, 0.02 and 0.05). The algorithm uses 1024-point FFT for

computing the identified model. The identified model (after 5 iterations) is given in Table 4.3.

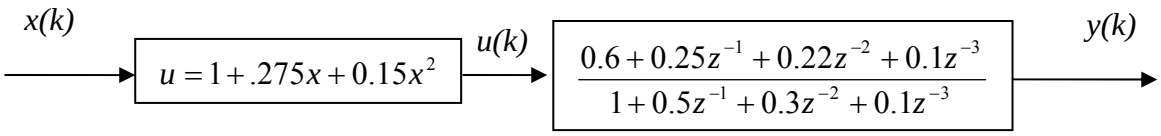

Figure 4.8: True Model, Example 2

Noise	**Nonlinear Block**	**Linear Block**	$\|Error\|_\infty$
0	$u = 1 + .275x + 0.15x^2$	$\frac{0.6 + 0.25z^{-1} + 0.22z^{-2} + 0.1z^{-3}}{1 + 0.5z^{-1} + 0.3z^{-2} + 0.1z^{-3}}$	0.0
0.01	$u = 1 + .275x + 0.15x^2$	$\frac{0.6 + 0.2495z^{-1} + 0.22z^{-2} + 0.0998z^{-3}}{1 + 0.4992^{-1} + 0.2999z^{-2} + 0.0997z^{-3}}$	0.0294
0.05	$u = 1 + .275x + 0.15x^2$	$\frac{0.6 + 0.235z^{-1} + 0.2211z^{-2} + 0.0927z^{-3}}{1 + 0.4749z^{-1} + 0.2998z^{-2} + 0.0906z^{-3}}$	0.1806

Table 4.2: Identified Model with Different Measurement Nose Level

(Example 2)

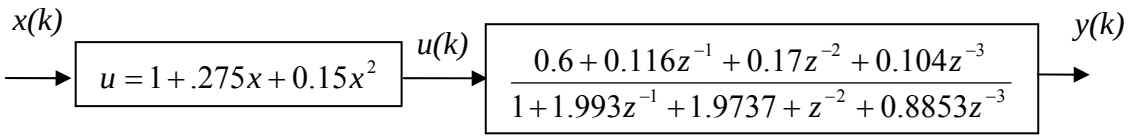

Figure 4.9: True Model, Example 3 (with poles very close to the Unit Cycle;

(-0.55+0.83i;-0.55-0.83i;-0.893)

Noise	**Nonlinear Block**	**Linear Block**	$\|Error\|_\infty$
0	$u = 1 + .275x + 0.15x^2$	$\frac{0.6 + 0.116z^{-1} + 0.17z^{-2} + 0.104z^{-3}}{1 + 1.993z^{-1} + 1.9737 + z^{-2} + 0.8853z^{-3}}$	0.0090
0.01	$u = 1 + .275x + 0.15x^2$	$\frac{0.6 + 0.116z^{-1} + 0.17z^{-2} + 0.104z^{-3}}{1 + 1.993z^{-1} + 1.9737 + z^{-2} + 0.8853z^{-3}}$	0.0464
0.05	$u = 1 + .275x + 0.15x^2$	$\frac{0.5999 + 0.116z^{-1} + 0.17z^{-2} + 0.104z^{-3}}{1 + 1.9929z^{-1} + 1.9736 + z^{-2} + 0.8853z^{-3}}$	0.8540

Table 4.3: Identified Model with Different Measurement Nose Level

(Example 3)

In Examples 1 and 2, the algorithm was tested with two different model orders. Looking at Table 4.1 and Table 4.2, It is observed that the algorithm produced comparable results to the true parameters values. However, in Example 3, the algorithm was tested against a system where its poles are very close to the unit cycle (-0.55+0.83i;-0.55-0.83i;-0.893), and the results in Table 4.3 indicates that the algorithm is very stable and produced very comparable to the true system parameters.

4.7 Concluding Remarks

In this chapter, a modified iterative procedure to identify Hammerstein models is developed. The modified algorithm was based on the work done in [11, 20]. The algorithm minimizes the H_∞ norm of the deviation between the true model and identified model. Illustrative examples were given to demonstrate the algorithm. It is also observed that the results are very close to the result reported in [11] and the algorithm is very stable in dealing with system with poles very close to the unit cycle.

CHAPTER 5

RECURSIVE IDENTIFICATION OF HAMMERSTEIN-WIENER (BAI'S SYSTEM) MODEL BY MINIMIZING THE H_∞ NORM OF THE MISMATCH ERROR

In this chapter, we propose a recursive identification algorithm to obtain a Hammerstein-Wiener model that minimizes the H_∞ Norm of the Mismatch error coupled with SVD to iteratively estimate the parameters.

This chapter is organized as follows: Section 1 will set the stage to the nonlinear system that will be used to develop the new algorithm. Then in Section 2, we will present the proposed H_∞ based algorithm and the notation used. Numerical examples are given in Section 3. Finally concluding remarks will be highlighted in section 4.

5.1 Introduction

Equation (3.1) which represents a nonlinear scalar stable discrete time dynamic system is used throughout this chapter. Refer to Section 3.3 for more details about Bai's system and the Two-Stage Identification Algorithm.

Al-Amer and Al-Sunni[11] have developed an algorithm to minimize the H_∞ Norm of the Mismatch Error, and Farooq [2] coupled the Recursive Least Square (RLS) identification scheme with Bai [1] Two Stage methods to come up with a Recursive Identification of the Bai's[1] system. The work here is an extension to the work in Chapter 4 and the work done by both Al-Amer and Al-Sunni [11] using the frequency domain and SVD to come up with a Recursive Least Square (RLS) algorithm to identify Hammerstein-Wiener nonlinear system

5.2 The Proposed Identification Algorithm

The system under consideration is shown in Figure 3.1. It consists of Hammerstein-Wiener models where two static nonlinear elements $N_1(.)$ and $N_2(.)$ surround a linear block $G(z)$ (Figure 3.1). The model is shown as a equation (3.1).

A set of N pairs of input and output data *{x(k),y(k)}* of the nonlinear system are assumed to be available. The system to be identified is a nonlinear single-input-single-output system that is modeled using equation (3.1) model. The $g_i(.)'s$ and $f_i(.)'s$ are nonlinear functions and $a = (a_1,.....,a_p)^{'}$, $b = (b_1,....,b_n)^{'}$

$c = (c_1,.....,c_m)^{'}$, $d = (d_1,.....,d_q)^{'}$ denote the system parameter vectors. The above system is known to be Hammerstein-Wiener nonlinear system [1].

The proposed algorithm is developed based on the Two-Stage identification algorithm developed by Bai [1] and Section 3.3.

In this chapter, we are working with the frequency domain of the model and system to be identified. Therefore let us define the following:

Let $U(z), Y(z), \text{ and } E(z)$ *denote the FFT (Fast Fourier Transform) of* $u(k), y(k), \text{ and } e(k)$ respectively.

We also have the following intermediate steps:

$$\begin{aligned}
&Y_F = FFT(y(k)) && k = 1,..., N \\
&U_F = FFT((u(k)) && k = 1,..., N \\
&F_t = FFT(f_t[u(k)]) && t = 1,..., m \\
& && k = 1,....N \\
&G_l = FFT(g_l[y(k)]) && l = 1,..., l \\
& && k = 1,..., N \\
&E_F = FFT(e(k))
\end{aligned} \qquad (5.1)$$

Then equation (3.1) will look, in the frequency domain, as follows:

$$Y_F(e^{jwk}) = \sum_{i=1}^{p} a_i \left\{ \sum_{l=1}^{q} d_l G_l[Y_F(k-i)] \right\} + \sum_{j=1}^{n} b_j \left\{ \sum_{t=1}^{m} c_t F_t[U_F(k-j)] \right\} + E_F(k) \qquad (5.2)$$

Define the following:

$$\theta(N) = (b_1c_1,\cdots,b_1c_m,\cdots,a_pd_1,\cdots,a_pd_q)^T$$
$$= (\theta_1,\cdots,\theta_{nm},\theta_{nm+1},\cdots,\theta_{nm+pq})^T$$

$$\Theta_{bc} = \begin{pmatrix} b_1c_1, & b_1c_2 & \cdots & ,b_1c_m \\ b_2c_1, & b_2c_2 & \cdots & ,b_2c_m \\ \vdots & \vdots & \ddots & \vdots \\ b_nc_1 & b_nc_2 & \cdots & b_nc_m \end{pmatrix}$$

$$\Theta_{ad} = \begin{pmatrix} a_1d_1, & a_1d_2 & \cdots & ,a_1d_q \\ a_2d_1, & a_2d_2 & \cdots & ,a_2d_q \\ \vdots & \vdots & \ddots & \vdots \\ a_pd_1 & a_pd_2 & \cdots & a_pd_q \end{pmatrix}$$

$$\phi_F(k) = (F_1[u(k-1)],\cdots,F_m[u(k-1)],\cdots,F_1[u(k-n)],\cdots,G_1[u(k-p)],\cdots,G_q[u(k-p)])^T$$

(5.3)

The system equation (3.1) can now be written in frequency domain as

$$Y_F(k) = \phi_F{}^T(k)\theta + E_F(k) \qquad (5.4)$$

Let

$$Y_{FN} = (Y_F(1),Y_F(2),\cdots,Y_F(N))^T \qquad ,E_{FN} = (E_F(1),\cdots,E_F(N))^T$$
and
$$\Phi_{FN} = (\phi_F{}^T(1),\cdots,\phi_F{}^T(N)) \qquad (5.5)$$
then
$$Y_{FN} = \Phi_{FN}\,\theta + E_{FN}$$

In this chapter we will present an identification algorithm to obtain

$\hat{a} = (\hat{a}_1,.....,\hat{a}_p)'$, $\hat{b} = (\hat{b}_1,....,\hat{b}_n)'$, $\hat{c} = (\hat{c}_1,.....,\hat{c}_m)'$, $\hat{d} = (\hat{d}_1,.....,\hat{d}_q)'$ so that the

H_∞ norm of the mismatch error is minimized. More precisely the following problem is to be solved.

$$\min_{\hat{a}_p \hat{b}_n \hat{c}_m \hat{d}_q} \left\| Y_{FN}(z) - \hat{Y}_{FN}(z) \right\|_\infty \qquad (5.6)$$

5.3 Implementation

The algorithm uses samples of the Z-transform of the input and output on the unit circle. This is obtained by computing the N-point discrete Fourier transform of *y*(*k*) and u(k). The computed Fourier transforms denoted by *Y* (e^{jw_k}) and U(e^{jw_k}) are basically samples of *Y* (*z*) and U(*z*) on the unit circle at the frequencies $w_k = \frac{2\pi k}{N}$

In this section we presented a procedure to identify a Hammerstein-Wiener model (Bai's) from a set of input output data to solve the optimization problem in equation (5.6).

According to Lawson generalized Algorithm [47], the solution for the H_∞ can be found via Weighted H_2 optimization problem as follows:

$$\left\| G_1 - G_2 \right\|_\infty = \left\| (G_1 - G_2) W \right\|_2$$

Where W is the weight matrix.

That is H_∞ approximation problem is obtained by a weighted least squares problem in the frequency domain. Each iteration of the algorithm involves solving a weighted least squares problem to find the augmented set of parameters:

$$\hat{\theta}(N) = (\hat{b}_1\hat{c}_1, \cdots, \hat{b}_1\hat{c}_m, \cdots, \hat{a}_p\hat{d}_1, \cdots, \hat{a}_p\hat{d}_q)^T$$

and

$$\hat{\theta}(N) = (\hat{\theta}_1, \cdots, \hat{\theta}_{nm}, \hat{\theta}_{nm+1}, \cdots, \hat{\theta}_{nm+pq})^T$$

$$\min_{\hat{\theta}_1,\cdots,\hat{\theta}_{nm},\hat{\theta}_{nm+1},\cdots,\hat{\theta}_{nm+pq}} \left\| \left[Y_{FN}(e^{jwk}) - \hat{Y}_{FN}(e^{jwk}) \right] W(e^{jwk}) \right\|_2 \qquad (5.7)$$

The frequency weighting at each iteration is computed as the product of the previous weight and the error computed using the latest available model. An initial value of the weight is given by $W(e^{jw_k}) = 1, \ \forall \ \ k$.

As in [4], using the SVD to extract the actual estimated system parameters

$$\hat{a} = (\hat{a}_1, ..., \hat{a}_p)', \qquad \hat{b} = (\hat{b}_1, ..., \hat{b}_n)', \ \hat{c} = (\hat{c}_1, ..., \hat{c}_m)', \qquad \hat{d} = (\hat{d}_1, ..., \hat{d}_q)'$$

Let $\hat{\Theta}_{bc}(N) = \sum_{i=1}^{\min(n,m)} \sigma_i \mu_i \nu_i^T$, $\hat{\Theta}_{ad}(N) = \sum_{i=1}^{\min(p,q)} \delta_i \zeta_i \xi_i^T$, be their singular values decomposition (SVD) where

$\mu_i's(i=1,2,....,p), \nu_i's(i=1,2,...,m),\ \zeta_i's(i=1,2,...,p)\ and\ \xi_i's(i=1,2,...,q)$ are n, m, p, q-dimensional orthonormal vectors respectively.

Let s_μ denotes the sign of the first non-zero element of μ_1 and s_ζ denotes the sign of the first non-zero element of ζ_1. Define the estimate as follows:

$$\hat{b}(N) = s_\mu \mu_1,\ \hat{c}(N) = s_\mu \sigma_1 \nu_1,\ \hat{a}(N) = s_\zeta \zeta_1,\ \hat{d}(N) = s_\xi \delta_1 \xi_1$$

A summary of the proposed algorithm is given below.

5.4 The identification Algorithm

Considering the system in equation (5.1); given *m, n, p, q, N*, and *{*(u(*k*), *y*(*k*)), *k* = 1, 2, *..N.}*.

Step 0: compute the estimated system parameters

$\hat{a} = (\hat{a}_1,.....,\hat{a}_p)'$, $\hat{b} = (\hat{b}_1,....,\hat{b}_n)'$ $\hat{c} = (\hat{c}_1,.....,\hat{c}_m)'$, $\hat{d} = (\hat{d}_1,.....,\hat{d}_q)'$ using Bai's Two Stage Identification Algorithm in section 3.3

Step 1: set $l = 1,\quad w_k^l = 1$ for $k \in [1, N]$.

Step 2: compute the FFT of all input, output, and all nonlinear function as in equation (5.1)

Step 3: compute $\hat{\theta}(N) = \hat{\theta}_{ls}(N) = \left(\Phi_{FN}^T W \Phi_{FN}\right)^{-1} \Phi_{FN}^T W Y_{FN}$ from equation (5.7)

Where

$$\hat{\theta}(N) = (\hat{\theta}_1, \cdots, \hat{\theta}_{nm}, \hat{\theta}_{nm+1}, \cdots, \hat{\theta}_{nm+pq})^T$$

and

$$\mathrm{W} = \mathrm{diag}(\ \mathrm{W}^{\mathrm{l}}(1), \mathrm{W}^{\mathrm{l}}(2), \cdots, \mathrm{W}^{\mathrm{l}}(\mathrm{N})\ )$$

Step 4: Update the weight matrix using

$$W^{l+1} = \frac{W^l \left| Y_{FN} - \hat{Y}_{FN} \right|}{\max(\max(W^l), 0.01)}$$

Step 5: set $l = l + 1$ and go to step 3.

The algorithm is terminated after a fixed number of iteration and the identified model is selected as the one that gives the least error.

Step 6: Extract the parameter estimates using singular value decomposition, according to the following. This is exactly the same as the 2nd stage of Bai's method.

and let

$$\hat{\Theta}_{bc}(N) = \sum_{i=1}^{\min(n,m)} \sigma_i \mu_i \nu_i^T, \quad \hat{\Theta}_{ad}(N) = \sum_{i=1}^{\min(p,q)} \delta_i \zeta_i \xi_i^T,$$

be their singular values decomposition (SVD) where

$\mu_i's(i=1,2,....,p), \nu_i's(i=1,2,...,m),\ \zeta_i's(i=1,2,...,p)$ *and* $\xi_i's(i=1,2,...,q)$

are n, m, p, q-dimensional orthonormal vectors respectively.

let s_μ denotes the sign of the first non-zero element of μ_1 and s_ζ denotes the sign of the first non-zero element of ζ_1. Define the estimate as follows:

$$\hat{b}(N) = s_\mu \mu_1,\ \hat{c}(N) = s_\mu \sigma_1 \nu_1,\ \hat{a}(N) = s_\zeta \zeta_1,\ \hat{d}(N) = s_\xi \delta_1 \xi_1$$

5.5 Illustrative Example

Example 5.1: The example used in Bai [1] will be used to illustrate the H_∞ norm of the mismatch error parametric identification algorithm results.

Consider the following Hammerstein-Wiener nonlinear system:

$$y(k) = a_1(d_1 y(k-1) + d_2 \sin(y(k-1)) + b_1(c_1 u(k-1) + c_2 u^2(k-1)) + b_2(c_1 u(k-2) + c_2 u^2(k-2)) + \eta(k)$$

where

$a = (a_1) = 1,\ d = (d_1, d_2)^T = (0.5, 0.25)^T$

$b = (b_1, b_2)^T = (0.4472, -0.8944)^T,\quad c = (c_1, c_2)^T = (1,4)^T$

The following input will be used for simulation:

$$u(k) = 2\sin(2k) + 2\sin(4k) + 0.15\sin(6k) + 0.15\sin(8k) + 0.1\sin(10k)$$

and

$\eta(k)$ are i.i.d random variables uniformly in [-0.5,0.5]

We have run the Identification of Hammerstein-Wiener Models to minimize the H_∞ Norm of the Mismatch Error Algorithm, and estimated parameters for six various levels of noise (0.0, 0.25, 0.50, and 0.75) and we have the following results:

Noise Level	a1		b1		b2		c1	
	Bai	H_∞	Bai	H_∞	Bai	H_∞	Bai	H_∞
0.00	1.0000	1.0000	0.4472	0.4472	-0.8944	-0.8944	1.0000	1.0000
0.10	1.0000	1.0000	0.4472	0.4477	-0.8945	-0.8942	0.9991	1.0036
0.25	1.0000	1.0000	0.4472	0.4471	-0.8944	-0.8945	1.0032	1.0068
0.50	1.0000	1.0000	0.4467	0.4515	-0.8947	-0.8923	0.9960	1.0202
0.75	1.0000	1.0000	0.4611	0.4472	-0.8873	-0.8944	0.9922	1.1001

Table 5.1: Identified Model with Different Measurement Noise Level for both Bai TSA and H_∞ (example 5.1)

Noise Level	c2		d1		d2		Error**	
	Bai	H_∞	Bai	H_∞	Bai	H_∞	Bai	H_∞
0.00	3.9999	3.9999	0.5000	0.5000	0.2500	0.2500	0.0000	0.0000
0.10	3.9999	4.0001	0.5000	0.5015	0.2533	0.2331	0.4115	0.5347
0.25	3.9999	3.9960	0.4994	0.5032	0.2446	0.2364	0.8349	0.9927
0.50	3.9999	3.9930	0.5002	0.5032	0.2573	0.2983	2.2044	2.7621
0.75	3.9995	3.9780	0.5008	0.4896	0.2947	0.2054	3.5061	4.9483

** error between the true plant and the identified model.

Table 5.1: Continuation- Identified Model with Different Measurement Noise Level for both Bai and H_∞ (example 5.1)

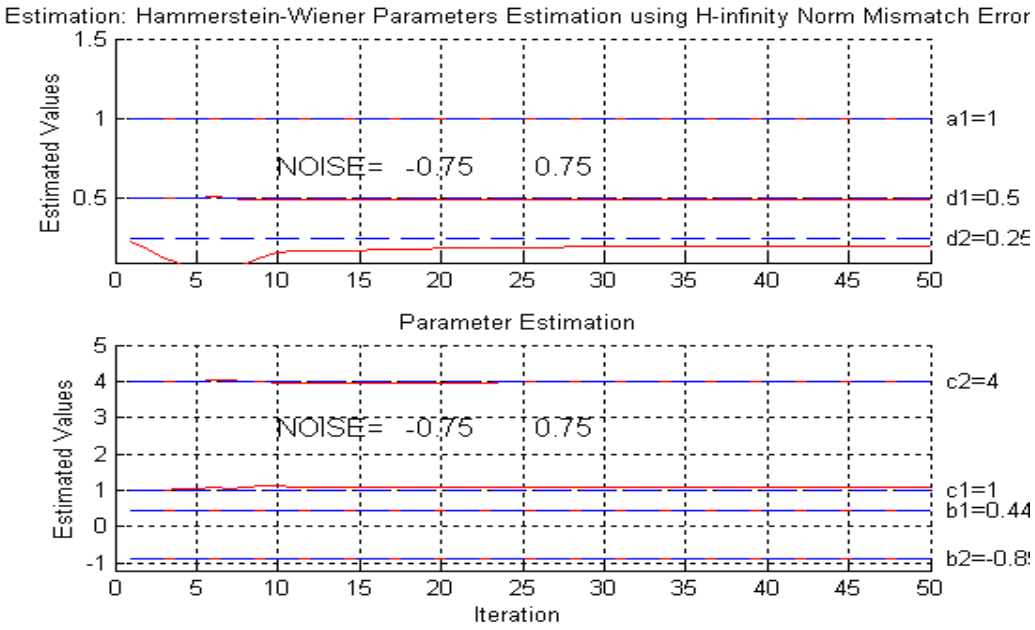

Figure 5.1: H_∞ Norm mismatch error for Bai's model ($\hat{a} = (\hat{a}_1,.....,\hat{a}_p)'$ $\hat{b} = (\hat{b}_1,....,\hat{b}_n)'$ & $\hat{c} = (\hat{c}_1,.....,\hat{c}_m)'$ $\hat{d} = (\hat{d}_1,.....,\hat{d}_q)'$) ; Ex. 5.1

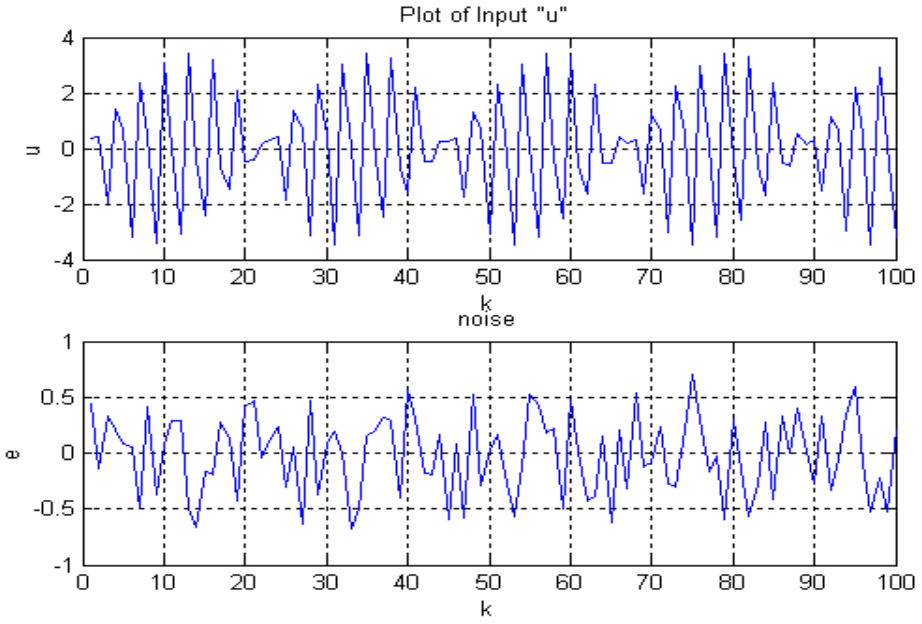

Figure 5.2: H_∞ Norm mismatch error for Bai's model (input and noise) for Ex. 5.1

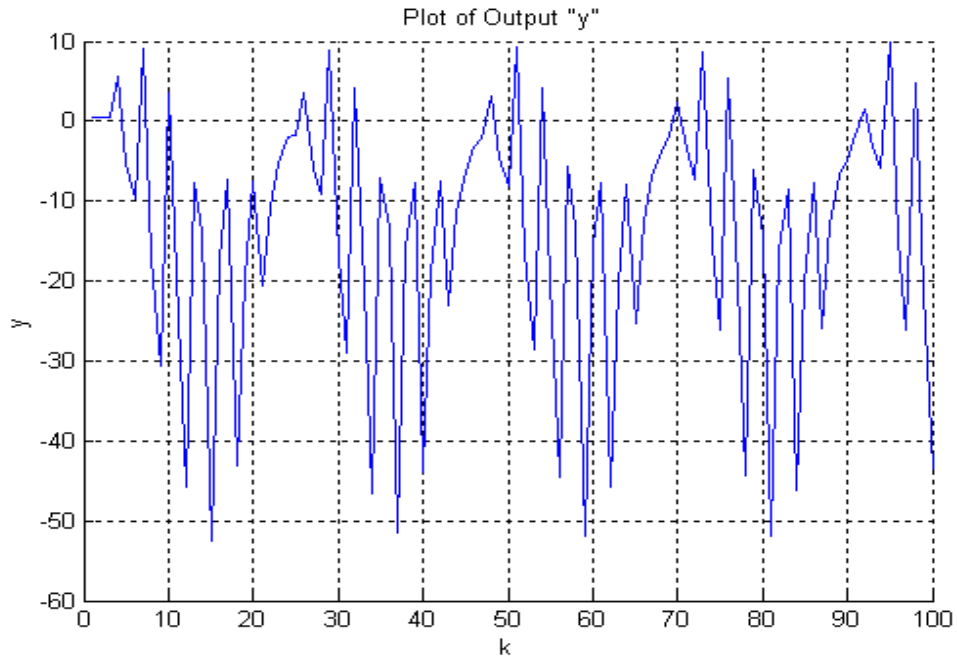

Figure 5.3: H_∞ Norm mismatch error for Bai's model (output) for example 5.1

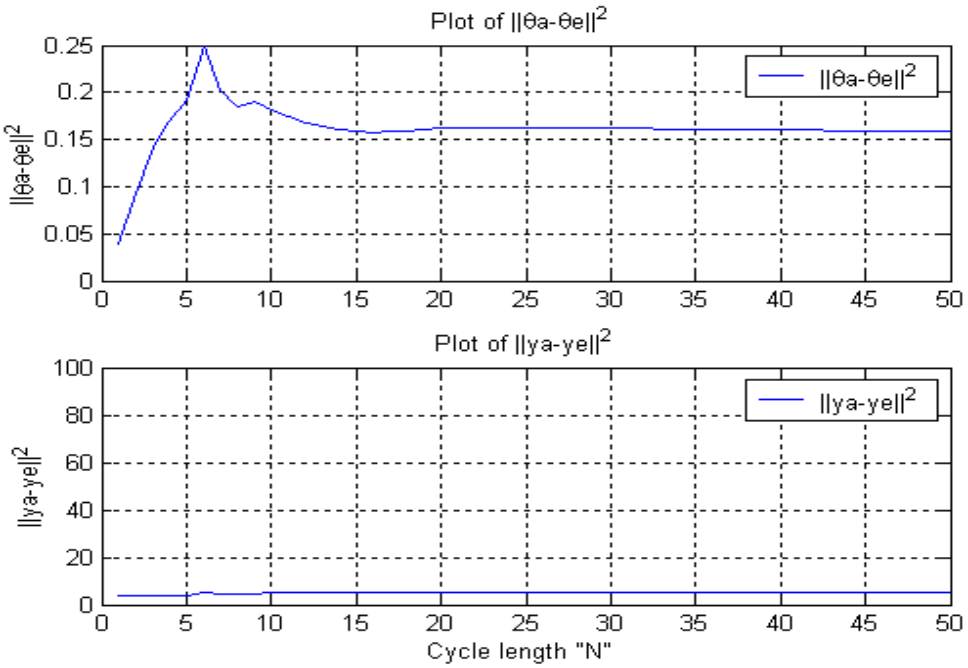

Figure 5.4: H_∞ Norm mismatch error for Bai's model ($\left\|\theta-\hat{\theta}\right\|$ *and* $\left\|Y-\hat{Y}\right\|$); Ex. 5.1

It can be seen from the above Table 5.1 and Figures 5.1 to 5.4 that the result of the H_∞ norm of the mismatch error identification algorithm is very encouraging and has produced very close result to the actual values of the original system parameters. In addition, the above table also lists the estimated system parameters for Bai's TSIA for the same example. It also includes both method errors between the nominal plant and the identified model.

In Figure 5.1, it is observed that almost all parameters converged to their best estimates within the 1st twenty (20) iteration, with exception of d_2 that took more iterations (30 iterations) to reach its final steady estimate. This behavior d_2 was also observed in the GA based identification algorithm.

In Figure 5.2, it is observed that the H_∞ norm of the mismatch error identification algorithm reached its largest error at iteration number six (6) and reached its minimum mismatch error around iteration number 33.

The example was also used to compare the performance of the Two Stage methodology and the H_∞ norm of the mismatch error identification algorithm to identify the parameters of the Hammerstein-Wiener nonlinear system. The table lists the objective functions (error between the plant and the identified model) of both the Two-Stage, and the proposed algorithms for Example 5.1. As expected, it shows that as the noise level increases, the error between the true system and the identified model increases

5.6 Concluding Remarks

An identification scheme for special class of non-linear systems was proposed. The equivalence between the two stage algorithm identification algorithm [1] developed in the literature for the identification of these systems was used to formulate the problem as Identification of Hammerstein-Wiener Models to minimize the H_∞ norm of the mismatch error which is converted to a weighted least square problem in the frequency domain. An example was illustrated to show the performance of the proposed algorithm. It is evident that the H_∞ norm of the mismatch error produced very encouraging parameter identification results that are very comparable result. However, it suggested that there should be more work to investigate the possibility to tune the algorithm's weighting equation to produce more accurate result than the result reported in both this work as well as in [1].

CHAPTER 6

RECURSIVE IDENTIFICATION OF HAMMERSTEIN-WIENER (BAI'S SYSTEM) MODEL MINIMIZING THE L_2 GAP

In This chapter, we proposed a recursive identification algorithm to obtain a Hammerstein-Wiener model that minimizes the L_2-gap error coupled with SVD to iteratively estimate the parameters.

6.1 Introduction

The gap metric was introduced to measure the closeness of two systems having the same feedback configurations. It is well known that two closed loop systems can be very closed even if the norm of the difference of the open loop blocks is arbitrarily high. For systems under feedback, it may be more appropriate to use the gap metric to measure the error

This chapter is organized as follows: Section 1 will be highlighting the history and a brief literature review of the L_2 -gap metric. Section 2, we will present the proposed L_2 -gap based algorithm and the notation used.

Numerical examples are given in Section 3. Finally concluding remarks will be highlighted in section 4.

6.2 Gap Metric

In this section, we first define the gap metric on the set of closed subspaces of a Hilbert space. This will then enable us to shortly review the notation of pointwise gap metric on the spaces of transfer functions. It will also serve as preliminary for the next sections, where we introduce the L_2 -gap metric.

In the following let H be a real complex Hilbert space with inner product <,> and let X,Y$\subset$ H be two closed subspaces. Denotes with P_X *and* P_Y the orthogonal projection on X respectively on Y.

Then the gap between the subspaces X and Y is defined by [48]:

$$gap(X,Y) := \left\| P_X - P_Y \right\| \tag{6.1}$$

Clearly gap(X,Y)=gap(Y,X) and gap(X,Y)=0 if and only if $P_X = P_Y$, i.e. X=Y.

Finally the triangle inequality:

$$gap(X,Z) \le gap(X,Y) + gap(Y,Z) \tag{6.2}$$

Lemma 6.1:[44]

Assume $x=(x_1,x_2,\cdots,x_q)^T$ and $y=(y_1,y_2,\cdots,y_q)^T$ are two non-zero (column) vectors in R^q and $X=span(x)$ and $Y=span(y)$, then one has

$$gap(X,Y)=\sin(x,y)=\frac{\|x\Lambda y\|}{\|x\|.\|y\|} \tag{6.3}$$

See [44] for the proof.

The gap metric was introduced by El-Sakkary and Zames [49] to study approximation and robustness of stability of systems with feedback interconnections. It captures the closeness of closed loop systems and is generally considered as very useful for the analysis of uncertain feedback systems. El-Sakkary [25] have proposed the following gap metric for SISO systems

$$\delta_L(P_1,P_2)=\sup_{\omega\in\Omega}\frac{\left|P_1(j\omega)-P_2(j\omega)\right|}{\sqrt{1+\left|P_1(j\omega)\right|^2}\sqrt{1+\left|P_2(j\omega)\right|^2}} \tag{6.4}$$

This is now known as the L_2-gap metric, and in general it is different from the gap metric. For the discrete time systems, the supremum is taken over the unit circle which leads to the following definition.

$$\delta_L(P_1,P_2)=\sup_{\omega\in[0,2\pi]}\frac{\left|P_1(e^{j\omega})-P_2(e^{j\omega})\right|}{\sqrt{1+\left|P_1(e^{j\omega})\right|^2}\sqrt{1+\left|P_2(e^{j\omega})\right|^2}} \tag{6.5}$$

6.3 Properties of Gap Metric

The gap metric is a metric that satisfies the following properties:

- $\delta_g(P_1, P_2) \geq 0 \quad with \quad \delta_g(P_1, P_2) = 0 \quad iff \quad P_1 = P_2$
- $\delta_g(P_1, P_2) = \delta_g(P_2, P_1)$
- $\delta_g(P_1, P_2) \leq \delta_g(P_1, P_3) + \delta_g(P_3, P_2)$

6.4 The Proposed Identification Algorithm

The system under consideration is shown in Figure 3.1. It consists of Hammerstein-Wiener models which may be considered as the system where two static nonlinear elements $N_1(.)$ and $N_2(.)$ surround a linear block $G(z)$. The model is shown is equation (3.4). Refer to Chapter 3 section 3.3 for more details.

A set of N pairs of input and output data *{x(k),y(k)}* of the nonlinear system are assumed to be available. The system to be identified is a nonlinear single-input-single-output system that will be modeled using equation (6.4) model. The $g_i(.)'s$ and $f_i(.)'s$ are nonlinear functions and $a = (a_1, ..., a_p)'$, $b = (b_1, ..., b_n)'$ $c = (c_1, ..., c_m)'$, $d = (d_1, ..., d_q)'$ denote the system parameter vectors. The above system is known to be Hammerstein-Wiener Nonlinear System [1].

Note that the parameters of the model can not be uniquely determined and the following uniqueness assumption shall be considered [1]:

- Assume that Θ_{ad} and Θ_{bc} are not both zero.

- Assume that $\|a\|_2 = 1$ *and* $\|b\|_2 = 1$ and the signs of the first non-zero elements of *a's* and *b's* are positive.

Also, let $U_F(z), Y_F(z)$, *and* $\eta_F(z)$ *denote the FFT (Fast Fourier Transform) of* $u(k), y(k)$, *and* $\eta(k)$ respectively.

We will have the following intermediate steps:

$$\begin{aligned}
&Y_F = FFT(y(k)) && k = 1,\ldots,N \\
&U_F = FFT((u(k)) && k = 1,\ldots,N \\
&F_t = FFT(f_t[u(k)]) && t = 1,\ldots,m \\
& && k = 1,\ldots N \\
&G_l = FFT(g_l[y(k)]) && l = 1,\ldots,l \\
& && k = 1,\ldots,N \\
&\eta_F = FFT(\eta(k)) &&
\end{aligned} \qquad (6.6)$$

Then equation (3.4) will look:

$$Y_F(e^{jwk}) = \sum_{i=1}^{p} a_i \left\{ \sum_{l=1}^{q} d_l G_l\left[Y_F(k-i)\right] \right\} + \sum_{j=1}^{n} b_j \left\{ \sum_{t=1}^{m} c_t F_t\left[U_F(k-j)\right] \right\} + \eta_F(k) \qquad (6.7)$$

Define the followings:

$$\begin{aligned}
\theta(N) &= (b_1c_1, \cdots, b_1c_m, \cdots, a_pd_1, \cdots, a_pd_q)^T \\
&= (\theta_1, \cdots, \theta_{nm}, \theta_{nm+1}, \cdots, \theta_{nm+pq})^T
\end{aligned}$$

$$\Theta_{bc} = \begin{pmatrix} b_1c_1, & b_1c_2 & \cdots & ,b_1c_m \\ b_2c_1, & b_2c_2 & \cdots & ,b_2c_m \\ \vdots & \vdots & \ddots & \vdots \\ b_nc_1 & b_nc_2 & \cdots & b_nc_m \end{pmatrix}$$

$$\Theta_{ad} = \begin{pmatrix} a_1d_1, & a_1d_2 & \cdots & ,a_1d_q \\ a_2d_1, & a_2d_2 & \cdots & ,a_2d_q \\ \vdots & \vdots & \ddots & \vdots \\ a_pd_1 & a_pd_2 & \cdots & a_pd_q \end{pmatrix}$$

$$\phi_F(k) = (F_1[U_F(k-1)],\cdots,F_m[U_F(k-1)],\cdots,F_1[U_F(k-n)],\cdots,G_1[Y_F(k-p)],\cdots,G_q[Y_F(k-p)])^T \quad (6.8)$$

The system in equation (6.4) can now be written in frequency domain as

$$Y_F(k) = \phi_F{}^T(k)\theta + \eta_F(k) \quad (6.9)$$

Where

$$Y_{FN} = (Y_F(1), Y_F(2), \cdots, Y_F(N))^T \qquad ,\eta_{FN} = (\eta_F(1), \cdots, \eta_F(N))^T$$

and

$$\Phi_{FN} = (\phi_F{}^T(1), \cdots, \phi_F{}^T(N))$$

then

$$Y_{FN} = \Phi_{FN}\theta + \eta_{FN}$$

In this chapter we presented an identification algorithm to obtain $a = (a_1,.....,a_p)^{'}$, $b = (b_1,....,b_n)^{'}$, $c = (c_1,.....,c_m)^{'}$, $d = (d_1,.....,d_q)^{'}$ so that the L_2-gap error is minimized. More precisely the following problem is to be solved.

$$\min_{\hat{a}_p\hat{b}_n\hat{c}_m\hat{d}_q} \delta_L\left(Y_{FN}(z) , \hat{Y}_{FN}(z) \right) \quad (6.10)$$

Computing the L_2-gap can be carried out using N samples of the frequency on the unit circle

$$\delta_L(P,G_r) \approx \max_{\omega_k \in [0,2\pi]} \frac{\left|P(e^{j\omega_k}) - G_r(e^{j\omega_k})\right|}{\sqrt{1+\left|P(e^{j\omega_k})\right|^2}\sqrt{1+\left|G_r(e^{j\omega_k})\right|^2}} \qquad (6.11)$$

The formula in equation (6.11) is a lower bound to the L_2-gap defined in equation (6.5) and they become closer as N increases.

6.5 Implementation

The algorithm uses samples of the Z-transform of the input and output on the unit circle. This is obtained by computing the N-point discrete Fourier transform of $y(k)$ and u(k). The computed Fourier transforms denoted by $Y(e^{jw_k})$ and $U(e^{jw_k})$ are basically samples of $Y(z)$ and $U(z)$ on the unit circle at the frequencies $w_k = \frac{2\pi k}{N}$

In this section we present a procedure to identify a Hammerstein-Wiener model (Bai's) from a set of input output data to solve the optimization problem in equation (6.11).

The proposed algorithm is based on the concept of the algorithm developed in Chapter 5**.** The solution to the L_2 approximation problem is obtained by solving a weighted least square problems in the frequency domain. Each iteration of the algorithm involves solving a weighted least squares problem to find the

augmented set of parameters $\hat{\theta}(N) = (\hat{b}_1\hat{c}_1, \cdots, \hat{b}_1\hat{c}_m, \cdots, \hat{a}_p\hat{d}_1, \cdots, \hat{a}_p\hat{d}_q)^T$ and $\hat{\theta}(N) = (\hat{\theta}_1, \cdots, \hat{\theta}_{nm}, \hat{\theta}_{nm+1}, \cdots, \hat{\theta}_{nm+pq})^T$

$$\min_{\hat{\theta}_1, \cdots, \hat{\theta}_{nm}, \hat{\theta}_{nm+1}, \cdots, \hat{\theta}_{nm+pq}} \left\| \left[Y_F(e^{jwk}) - \hat{Y}_F(e^{jwk}) \right] W(e^{jwk}) \right\|_2 \qquad (6.12)$$

The frequency weighting at each iteration is computed as the product of the previous weight and the error computed using the latest available model. An initial value of the weight is given by $W(e^{jw_k}) = 1, \ \forall \ \ k$.

Using the SVD to extract the actual estimated system parameters

$$\hat{a} = (\hat{a}_1, ..., \hat{a}_p)', \quad \hat{b} = (\hat{b}_1, ..., \hat{b}_n)', \ \hat{c} = (\hat{c}_1, ..., \hat{c}_m)', \quad \hat{d} = (\hat{d}_1, ..., \hat{d}_q)'$$

Let $\hat{\Theta}_{bc}(N) = \sum_{i=1}^{\min(n,m)} \sigma_i \mu_i \nu_i^T$, $\hat{\Theta}_{ad}(N) = \sum_{i=1}^{\min(p,q)} \delta_i \zeta_i \xi_i^T$, be their singular values decomposition (SVD) where

$\mu_i's (i = 1,2,...., p), \nu_i's (i = 1,2,..., m), \ \zeta_i's (i = 1,2,..., p)$ *and* $\xi_i's (i = 1,2,..., q)$ are n, m, p, q-dimensional orthonormal vectors respectively.

Let s_μ denotes the sign of the first non-zero element of μ_1 and s_ζ denotes the sign of the first non-zero element of ζ_1. Define the estimate as follows:

$$\hat{b}(N) = s_\mu \mu_1, \ \hat{c}(N) = s_\mu \sigma_1 \nu_1, \ \hat{a}(N) = s_\zeta \zeta_1, \ \hat{d}(N) = s_\xi \delta_1 \xi_1$$

A summary of the proposed algorithm is given below.

6.6 The identification Algorithm

Considering the System in equation (5.1) Given *m, n, p, q, N*, and *{*(u(*k*), *y*(*k*)), *k* = 1, 2, *..N.}*.

Step 0: compute the estimated system parameters $\hat{a} = (\hat{a}_1,.....,\hat{a}_p)^{'}$, $\hat{b} = (\hat{b}_1,....,\hat{b}_n)^{'}$ $\hat{c} = (\hat{c}_1,.....,\hat{c}_m)^{'}$, $\hat{d} = (\hat{d}_1,.....,\hat{d}_q)^{'}$ using Bai's Two Stage Identification Algorithm in Section 3.3

Step 1: set $l = 1$ and the weight to 1's for all k=1 to N.

Step 2: compute the FFT of all input, output, and all nonlinear function as in Equation (5.5)

Step 3: compute $\hat{\theta}(N) = \hat{\theta}_{ls}(N) = \left(\Phi F_N^T\, W\, \Phi F_N\right)^{-1} \Phi F_N^T\, W\, YF_N$ from Equations (5.7) and (5.8)

Where

$$\hat{\theta}(N) = (\hat{\theta}_1,\cdots,\hat{\theta}_{nm},\hat{\theta}_{nm+1},\cdots,\hat{\theta}_{nm+pq})^T$$

and

$$\mathrm{W} = \mathrm{diag}(\ \mathrm{W}^{l}(1),\mathrm{W}^{l}(2),\cdots,\mathrm{W}^{l}(\mathrm{N})\ )$$

Step 4: Update the weight using

$$W^{l+1}(\omega_{\mathrm{k}}) = \frac{W^{l}(\omega_{\mathrm{k}})}{\alpha}\ \frac{\left|Y_F(e^{jw_k}) - \hat{Y}_F(e^{jw_k})\right|}{\sqrt{1+\left|Y_F(e^{jw_k})\right|^2}\sqrt{1+\left|\hat{Y}_F(e^{jw_k})\right|^2}}$$

We select $\alpha = \frac{1}{\sqrt{1+\left|Y_F(e^{jw_k})\right|^2}\sqrt{1+\left|\hat{Y}_F(e^{jw_k})\right|^2}}$

Step 5: set $l = l+1$ and go to step 3.

The algorithm is terminated after a .fixed number of iteration and the identified model is selected as the one that gives the least error.

Step 6: Extract the parameter estimates using singular value decomposition, according to the following. This exactly the same as the 2nd stage of Bai's method.

And let $\hat{\Theta}_{bc}(N) = \sum_{i=1}^{\min(n,m)} \sigma_i \mu_i v_i^T$, $\hat{\Theta}_{ad}(N) = \sum_{i=1}^{\min(p,q)} \delta_i \zeta_i \xi_i^T$, be their singular values decomposition (SVD) where

$\mu_i's(i = 1,2,...., p), v_i's(i = 1,2,..., m),\ \zeta_i's(i = 1,2,..., p)$ *and* $\xi_i's(i = 1,2,..., q)$

are n, m, p, q-dimensional orthonormal vectors respectively.

let s_μ denotes the sign of the first non-zero element of μ_1 and s_ζ denotes the sign of the first non-zero element of ζ_1. Define the estimate as follows:

$\hat{b}(N) = s_\mu \mu_1,\ \hat{c}(N) = s_\mu \sigma_1 v_1,\ \hat{a}(N) = s_\zeta \zeta_1,\ \hat{d}(N) = s_\xi \delta_1 \xi_1$

6.7 Illustrative Example

Example 6.1: The example used in Bai [1] will be used to illustrate the GA Parametric Identification algorithm results.

Consider the following Hammerstein-Wiener nonlinear system:

$$y(k) = a_1(d_1 y(k-1) + d_2 \sin(y(k-1)) + b_1(c_1 u(k-1) + c_2 u^2(k-1)) + b_2(c_1 u(k-2) + c_2 u^2(k-2)) + \eta(k)$$

where

$$a = (a_1) = 1,\ d = (d_1, d_2)^T = (0.5, 0.25)^T$$

$$b = (b_1, b_2)^T = (0.4472, -0.8944)^T,\quad c = (c_1, c_2)^T = (1,4)^T$$

for simulation, the following input will be used:

$$u(k) = 2\sin(2k) + 2\sin(4k) + 0.15\sin(6k) + 0.15\sin(8k) + 0.1\sin(10k)$$

and

$\eta(k)$ are i.i.d random variables uniformly in [-0.5,0.5]

We have ran the Identification of Hammerstein-Wiener Models to minimize the L_2 Norm of the Mismatch Error Algorithm, and estimated parameters for six various levels of noise (0.0, 0.1, 0.25, 0.5, and 0.75) and we have the following results:

Noise Level	a1		b1		b2		c1	
	Bai	L_2	Bai	L_2	Bai	L_2	Bai	L_2
0.00	1.0000	1.0000	0.4472	0.4472	-0.8944	-0.8944	1.0000	1.0000
0.10	1.0000	1.0000	0.4472	0.4473	-0.8944	-0.8944	0.9997	0.9958
0.25	1.0000	1.0000	0.4472	0.4471	-0.8944	-0.8945	0.9997	0.9701
0.50	1.0000	1.0000	0.4472	0.4471	-0.8944	-0.8944	1.0004	1.0681
0.75	1.0000	1.0000	0.4473	0.4355	-0.8944	-0.9002	0.9998	1.0289
Noise Level	**c2**		**d1**		**d2**		**Error**	
	Bai	L_2	**Bai**	L_2	**Bai**	L_2	**Bai**	L_2
0.00	3.9999	3.9997	0.5000	0.5000	0.2500	0.2500	0.0011	0.0014
0.10	4.0000	4.0007	0.5000	0.5008	0.2535	0.2461	1.4996	1.9026
0.25	4.0000	4.0022	0.5000	0.5016	0.2550	0.2492	3.2317	4.0256
0.50	3.9998	4.0066	0.5000	0.5029	0.2450	0.2704	6.3537	7.9315
0.75	4.0005	4.0115	0.4999	0.5048	0.2611	0.2501	9.9939	12.386

Table 6.1: Identified Model with Different Measurement Noise Level for both Bai and L_2 (example 1)

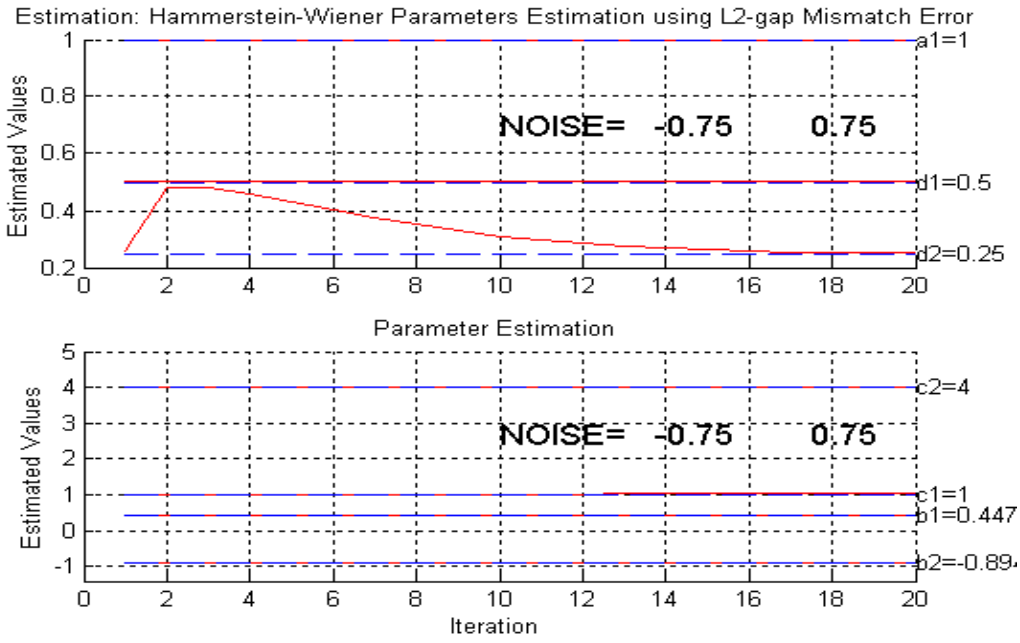

Figure 6.1: $L_2 - gap$ error for Bai's model ($\hat{a} = (\hat{a}_1,.....,\hat{a}_p)'$, $\hat{b} = (\hat{b}_1,....,\hat{b}_n)'$, $\hat{c} = (\hat{c}_1,.....,\hat{c}_m)'$ & $\hat{d} = (\hat{d}_1,.....,\hat{d}_q)'$) Ex. 6.1

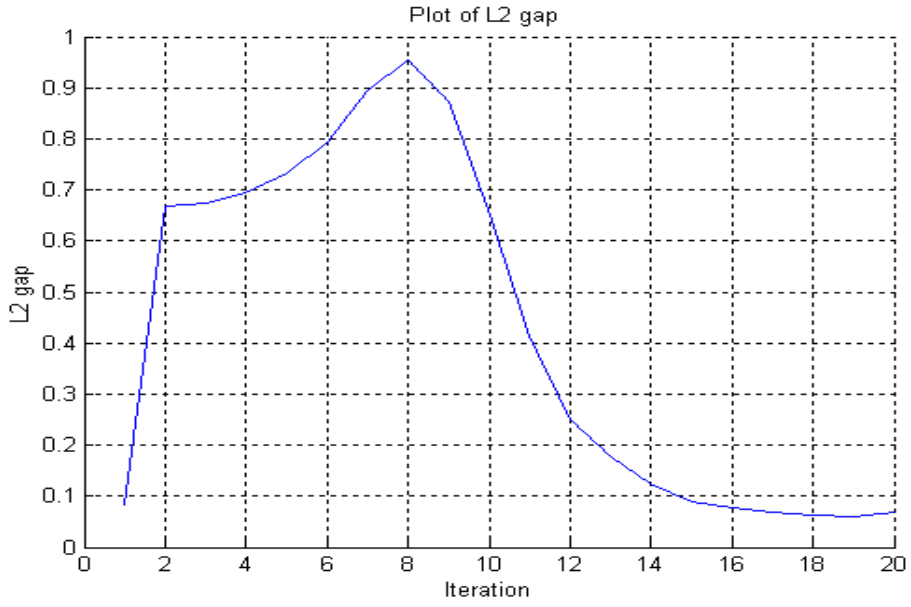

Figure 6.2: $L_2 - gap$ error for Bai's model for example 6.1

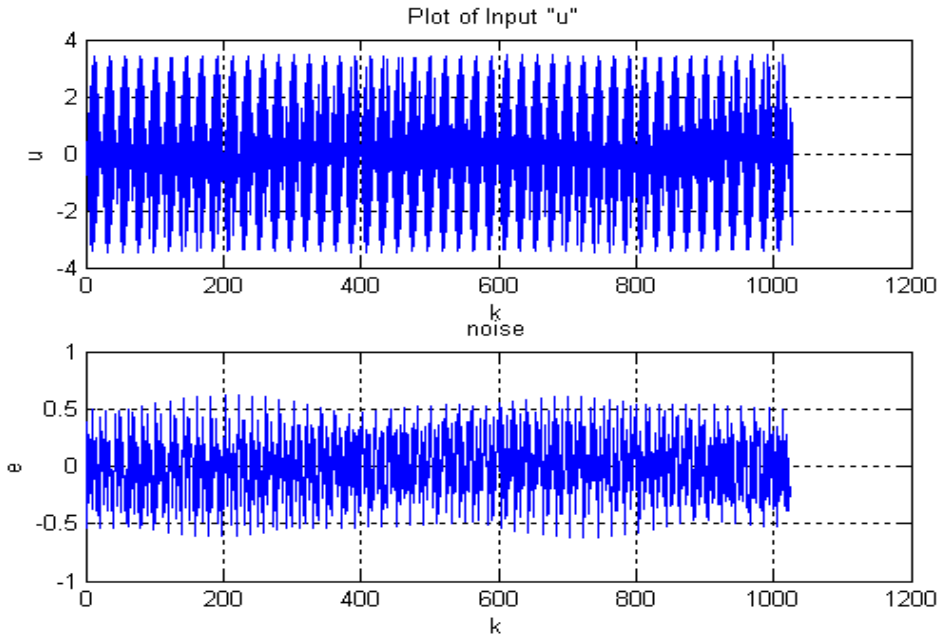

Figure 6.3: $L_2 - gap$ error for Bai's model (input and noise) Ex. 6.1

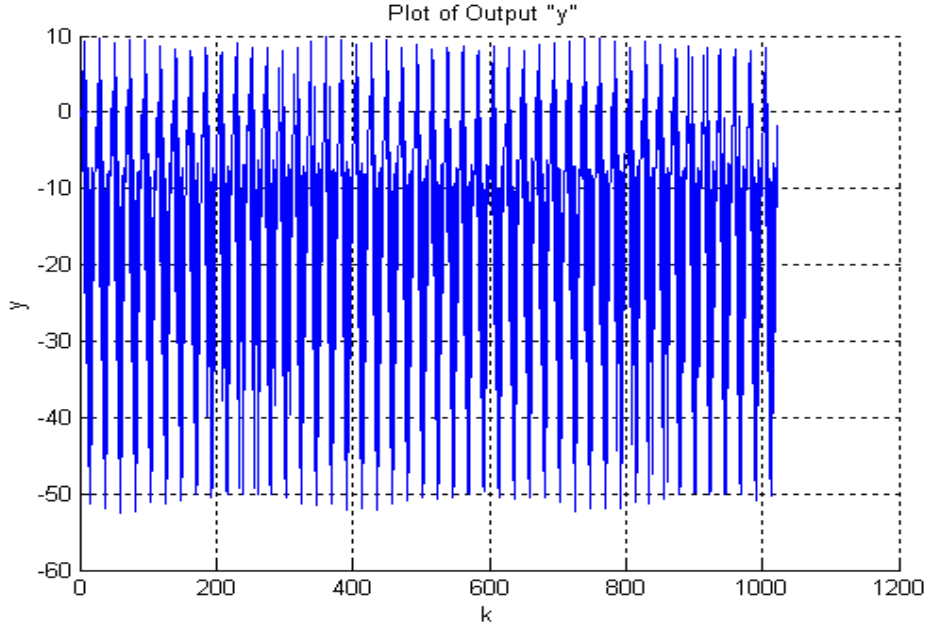

Figure 6.4: $L_2 - gap$ error for Bai's model (output) for example 6.1

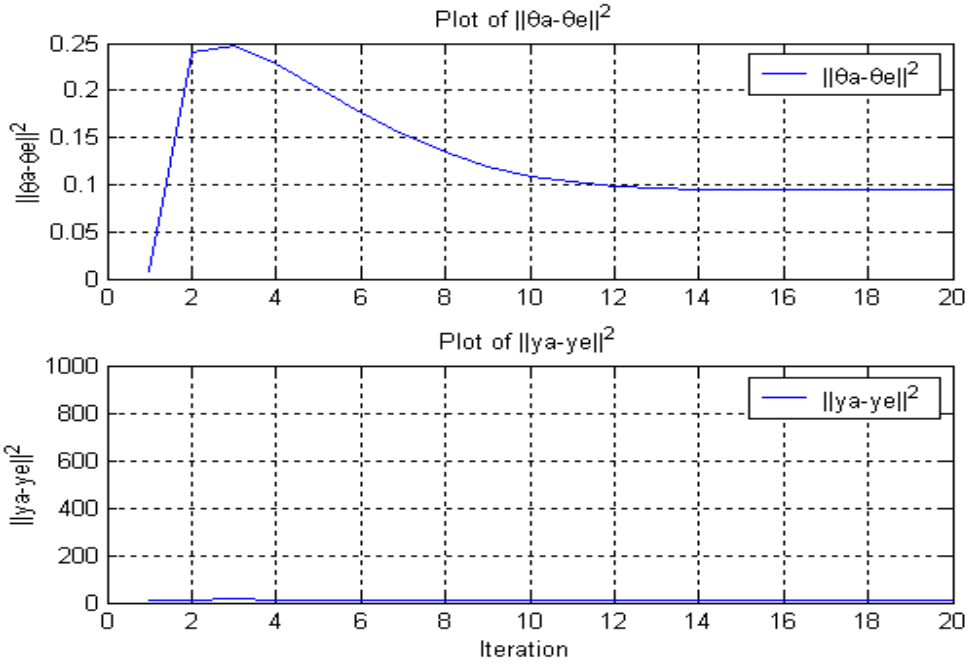

Figure 6.5: $L_2 - gap$ error for Bai's model ($\left\|\theta - \hat{\theta}\right\|$ and $\left\|Y - \hat{Y}\right\|$) for example 6.1

It can be seen from Table 6.1 and Figures 6.1, 6.2 and 6.5 that the result of the L_2 -gap error based identification algorithm is very encouraging and has produced very close results to the actual values of the original system parameters. In addition, Table 6.1 also lists the estimated system parameters for Bai's TSIA for the same example. It also includes, for both methods, the errors between the nominal plant and the identified model.

In Figure 6.1, it is observed that almost all parameters converged to their best estimates within the second iteration, with exception of d_2 that took more iterations (20 iterations) to reach to its final steady estimate. This behavior of parameter d_2 was also observed in both the GA based identification algorithm and the H_∞ norm mismatch error identification algorithm.

In Figure 6.2, it is observed that the H_∞ norm of the mismatch error identification algorithm reached its lowest L_2 -gap error at iteration number nineteen (19) and this iteration's parameters values were considered the best estimate that minimizes the L_2 -gap error.

The example was also used to compare the performance of the Two Stage methodology and the L_2 -gap error identification algorithm to identify the parameters of the Hammerstein-Wiener nonlinear system. Table 6.1 is listing the objective functions (error between the plant and the identified model) of both the Two-Stage, and the proposed algorithms for Example 6.1. As expected, it shows

that as the noise level increase, the error between the true system and the identified model increase.

In Figure 6.5, it is observed that the system output error between the nominal system and the identified models reached its peak at iteration number five (5) and started to decrease to reach its final value around iteration nineteen and twenty. This behavior is due to the behavior of parameter d_2 in Figure 6.1.

6.8 Concluding Remarks

An identification scheme for special class of non-linear systems is proposed. The equivalence between the two stage identification algorithm developed in the literature [1] for the identification of these systems was used to formulate the problem as identification of Hammerstein-Wiener models to minimize the L_2 gap error which is converted to a weighted least squares problem in the frequency domain. An example was illustrated to test the performance of the proposed algorithm. It is evident that the L_2 error based algorithm produces very good results compared to both the Two-Stage Identification Algorithm and the true parameters values. However, we have noticed that when we choose α other than the suggested value the algorithm's performance become very erratic and unpredictable.

CHAPTER 7

IDENTIFICATION OF HAMMERSTEIN MODEL MINIMIZING THE ν-GAP METRIC

In This chapter, we report an initial result of the development of a recursive identification algorithm to obtain a Hammerstein model that minimizes the $\nu - gap$ coupled with SVD to iteratively estimate the parameters.

7.1 Introduction

In this section, we will start with defining what $\nu - gap$ is, and how we can use it to identify nonlinear systems (Hammerstein).

Let us first define the following notation [22]:

Let **R** and **C** denote the set of real and complex numbers respectively.

Let ***D*** denote the open unit disk, $D := \{z : |z| < 1\}$.

Let ∂D denote the boundary of ***D.***

Let L_∞ denotes the normed space of all functions essentially bounded on ∂D and having norm $\|f\|_{L_\infty} := ess\sup_\omega \overline{\sigma}(f(e^{i\omega}))$

Let us first define the $\nu - gap$ [19]:

$$\delta_\nu(P_1,P_2) = \begin{cases} \inf\limits_{Q,Q^{-1}\in L_\infty} \|G_1 - G_2 Q\|_\infty & ;\ if \quad I(P_1,P_2) = 0 \\ 1 \qquad otherwise \end{cases} \tag{7.1}$$

Where

$I(P_1,P_2) := wno\ \det(G_2^* G_1)$

where $wno(g)$ denotes the winding number of $g(z)$ evaluated on the standard Nyquist contour indented around any poles on ∂D.

$\nu - gap$ is a metric and therefore all properties mentioned in Chapter 6 are properties of the $\nu - gap$ as well. The ν-gap metric has well established properties that are useful in the study of robust control systems. We can say that if $\delta_\nu(P_1,P_2)$ is small then any satisfactory controller for P_1 will also be satisfactory for P_2 [34]. Also if the $\delta_\nu(P_1,P_2)$ is large then there are satisfactory controller for P_1 which perform badly with P_2, and vice versa, because δ_ν is a metric, satisfying

$\delta_\nu(P_1,P_2)=\delta_\nu(P_2,P_1)$. Properties of ν-gap are extensively studied by Vinnicombe [26].

7.2 Identification in the $\nu-gap$ metric

A logical objective to the parametric identification of nonlinear system is to minimize the $\nu-gap$ between the true plant and the model. El-Sakkary [16] have proposed the following gap metric for SISO systems

$$\delta_L(P_1,P_2)=\sup_{\omega\in\Omega}\frac{\left|P_1(j\omega)-P_2(j\omega)\right|}{\sqrt{1+\left|P_1(j\omega)\right|^2}\sqrt{1+\left|P_2(j\omega)\right|^2}} \tag{7.2}$$

This is known as the L_2-gap metric, and a closely related gap metric is the $\nu-gap$ metric which is defined as [35].

$$\delta_\nu(P_1,P_2)=\begin{cases}\delta_L(P_1,P_2) & \text{if } 1+P_2^*(j\omega)P_1(j\omega)\neq 0 \ \ \forall\omega \ \text{ and} \\ & wno(1+P_2^*P_1)+\eta(P_1)-\eta(P_2)=0 \\ 1 & otherwise\end{cases} \tag{7.3}$$

where $\eta(P)$ and *wno(P)* denote the number of right half plane poles and winding number of *P* respectively.

An algorithm was proposed for the identification of linear SISO systems using the ν-gap metric. The algorithm involves solving a series of LMI

optimization problems followed by Hankel approximation [22]. Al-Amer [12], proposed an iterative weighted least squares algorithm to solve the L_2-gap metric identification problem for linear systems. The use of ν-gap for measuring uncertainty in nonlinear system was presented in [24].

We facilitate the use of the $\nu - gap$ in identifying Hammerstein nonlinear systems, we sated the following assumptions:

1. The nonlinear block (N) structure and order of the identified model are known.
2. The order of the linear model of Hammerstein is not known.
3. Both original system and identified model are stable.

7.3 The Proposed Identification Algorithm

The proposed algorithm to identify Hammerstein nonlinear system consists of the following steps:

Step 1: Assume the linear part of the identified models of certain order.

Step 2: Identify the Hammerstein model using the L_2-gap error as stated in previous chapter.

Step 3: Check if the $\nu - gap$ condition stated in equation (7.3) is satisfied. If satisfied, check the error between the true system and the identified

model. If not close enough, increase the model order and go back to step 2.

7.4 Illustrative Examples

The nonlinear system being identified is shown in Figures 7.1 and 7.2. The true Hammerstein nonlinear system has order of n = 3, m = 3. The input x(k) is generated as a uniformly distributed sequence of magnitude 1. The measurement noise is uniformly distributed with noise level 0.05. The identified model (after 20 iterations) is given in Table 7.1.

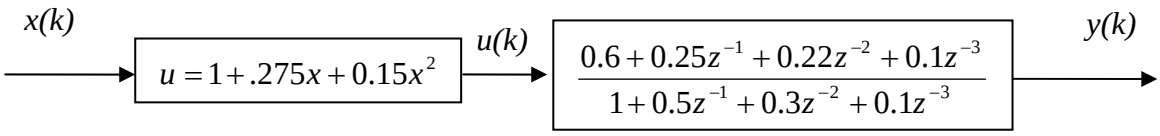

Figure 7.1: True Model example 1

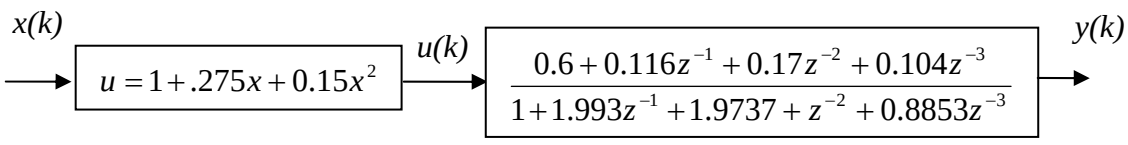

Figure 7.2: True Model, Example 2 {with poles very close to the Unit Cycle (-0.55+0.83i;-0.55-0.83i;-0.893) }

Model Order (n,m)	**Linear Block**	ν-gap	Cond.1: $1+P_2^*(j\omega)P_1(j\omega)\neq 0 \ \forall\omega$ $wno(1+P_2^*P_1)+\eta(P_1)-\eta(P_2)=0$ $\min(\mathrm{Re}(cond\,1))$ * $\max(\mathrm{Re}(cond\,1)>0$
n=1 m=1	$\frac{0.6035+0.1296z^{-1}}{1+0.298z^{-1}}$	0.8786	1.6349 > 0 0.9960 > 0
n=2 m=2	$\frac{0.6006-0.0508z^{-1}+0.2892z^{-2}}{1-0.0012z^{-1}+0.3731z^{-2}}$	0.9987	3.9504 > 0 satisfied -3.4855 < 0 not satisfied
n=3 m=3	$\frac{0.6+0.2494z^{-1}+0.2201z^{-2}+0.0997z^{-3}}{1+0.4900z^{-1}+0.3001z^{-2}+0.0996z^{-3}}$	0.0024	3.4694 > 0 satisfied 3.4873 > 0 satisfied

Table 7.1: Identified Model with different model order using ν-gap based identification algorithm for Example 1

Model Order (n,m)	**Linear Block**	ν-gap	Cond.1: $1+P_2^*(j\omega)P_1(j\omega)\neq 0 \ \forall\omega$ $wno(1+P_2^*P_1)+\eta(P_1)-\eta(P_2)=0$ $\min(\mathrm{Re}(cond\,1))$ * $\max(\mathrm{Re}(cond\,1)$
n=3 m=3	$\frac{0.6+0.116z^{-1}+0.17z^{-2}+0.104z^{-3}}{1+1.993z^{-1}+1.9737+z^{-2}+0.8853z^{-3}}$	0.0	1.3599 > 0 satisfied 1.3664 > 0 satisfied
n=1 m=2	$\frac{0.3031 \ -0.2233z^{-1}}{1+1.1405z^{-1}+0.9505z^{-2}+0.49z^{-3}}$	0.5145	1.0079 > 0 satisfied 0.9746 > 0 satisfied
n=2 m=3	$\frac{0.7078+0.0888z^{-1}+0.2348z^{-2}}{1+1.9256z^{-1}+1.9007z^{-2}+0.8200z^{-3}}$	0.5359	1.0337 > 0 satisfied 1.0137 > 0 satisfied
n=3 m=4	$\frac{0.5999+0.1146z^{-1}+0.1698z^{-2}+0.1035z^{-3}}{1.0000 \ \ 1.9907z^{-1} \ \ 1.9693z^{-2}+0.8810z^{-3} \ \ -0.0019z^{-4}}$	0.9981	1.0967 > satisfied -1.0967 < 0 not satisfied

Table 7.2: Identified Model using ν-gap based identification algorithm - Example 3 (with poles very close to the Unit Cycle;(-0.55+0.83i;-0.55-0.83i;-0.893)

In Table 7.1 and 7.2, it is observed that when the conditions are satisfied and the ν-gap is less than 0.3, it can be said that the true system and the identified model are very close and can be used for other control system purposes.

It is also noticed that when the conditions are not satisfied then, nothing can be said about the identified model and further analysis needs to be done to check it ν-gap closeness.

In Table 7.2, it is observed that the identified algorithm was able to deal with systems with poles very close to the unit cycle.

7.5 Concluding Remarks

A logical aim for identification schemes is to minimize the ν-gap between the true plant and the model. We can say that the identified model has mimicked the true system and with small $\delta_\nu(P_1, P_2)$. The identified model satisfactory controller will also be satisfactory for the true plant. In addition, we can determine the identified model order by minimizing the $\delta_\nu(P_1, P_2)$ and iteratively increasing the order till we get small $\delta_\nu(P_1, P_2)$. Finally, using the $\delta_\nu(P_1, P_2)$ will give us more information about the robustness of the identified plant.

CHAPTER 8

SCHEMES PERFORMANCE ANALYSIS

In this chapter, the performance of all developed identification algorithms will be compared.

8.1 Performance Analysis Factors

The following factors were considered in the comparatives performance analysis:

- The error between the true system and the identified model.
- The number of iterations/generations that took each parameter to get to the final stable value.
- The stability of the developed identification scheme.

8.2 Comparative Analysis Tables

The following tables are collections of all required data that was analyzed and the performances of all newly developed identification algorithms is compared. The data gathered is for the case where noise +/- 0.50

Scheme	a1	%error	b1	%error	b2	%error	c1	%error
H_∞	1.000	0.00%	0.4472	0.00%	-.8944	0.00%	1.1001	10.0%
$L_2 - gap$	1.000	0.00%	0.4355	2.62%	-.9002	0.65%	1.0289	2.3%
GA	1.000	0.0%	0.4473	.002%	-.8944	0.00%	1.0002	0.02%
	c2	%error	d1	%error	d2	%error	ERROR	
H_∞	3.9780	0.55%	0.4896	2.00%	0.2054	17.8%	4.948	
$L_2 - gap$	4.0115	0.28%	0.5048	0.26%	0.2501	0.04%	12.386	
GA	3.9994	.002%	0.5000	0.00%	0.2524	0.96	9.0327	

Table 8.1: Comparative estimated parameters and their associated error %

	a1	ITR#	b1	ITR#	b2	ITR#	c1	ITR#
H_∞	1.000	1	0.4472	1	-.8944	1	1.1001	11
$L_2 - gap$	1.000	1	0.4355	1	-.9002	1	1.0289	1
GA	1.000	1	0.4473	28	-.8944	28	1.0002	77
	c2	**ITR#**	**d1**	**ITR#**	**D2**	**ITR#**	**ERROR**	
H_∞	3.9780	27	0.4896	7	0.2054	27	4.948	
$L_2 - gap$	4.0115	1	0.5048	1	0.2501	17	12.386	
GA	3.9994	32	0.5000	82	0.2524	75	9.0327	

Table 8.2: Comparative estimated parameters and their associated final value vs. number of iterations to reach stable estimated value.

8.3 Performance Analysis

We have analyzed the tables in section 8.2 and the figures in Chapter 4, 5, and 6 and concluded the following performance analysis of the newly developed identification algorithms:

- GA based algorithm is more stable than others
- $L_2 - gap$ error algorithm converges faster to the accurate parameter estimates than GA and the H_∞ norm of the mismatch error.
- H_∞ norm of the mismatch error algorithm produces more accurate parameter estimates than GA and $L_2 - gap$.
- H_∞ and $L_2 - gap$ based algorithms produce less output error than GA.

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

This chapter concludes the thesis by presenting the conclusions, summarizing important contributions and highlights some of the future research work that can be further investigated by other researchers and Master of Science degree candidates

9.1 Conclusions

Nonlinear system identification via Hammerstein-Wiener model representations have been considered in this research. Unlike alternative approaches that are mainly the traditional iterative algorithm proposed by Narendra and Gallman [9] and correlation techniques in [15-18], the more recent approach for the identification of Hammerstein-Wiener systems has been introduced by Bai [1]. This algorithm is based on least squares estimation (LSE) and singular value decomposition (SVD), however it only applies to the single-

input/single-output (SISO) case, and consistency of the estimates can only be assured for the case of the disturbances being white noise, or in the noise-free case [1].

The contribution of the Thesis is summarized by the following:

1. Developed a new algorithm for parameters identification of Hammerstein-Wiener and Bai [1] model. The two stage algorithm of Bai [1] was formulated in such a way so that GA can be used to solve it. Through two examples, the proposed GA based identifications scheme has produced very encouraging results with the estimated parameters very close to the true values of the original system parameters. It also produced results that are very close to the results produced by Bai's Two Stage identification algorithm.

2. Developed a modified iterative procedure to identify Hammerstein models. The modified algorithm was based on the work done in [19, 20]. The algorithm is to minimize the H_∞ Norm of the mismatch error between the true model and identified model. Illustrative examples were given that have demonstrated the algorithm. It is observed that the results are very close to the result reported in [11].

3. Developed an iterative frequency domain based algorithm to identify Hammerstein-Wiener models to minimize the H_∞ Norm of the mismatch error between the true model and identified model. This work based on Bai's Two Stage Identification algorithm [1], Lawson generalized Algorithm [47], and Al-Amer and Al-Sunni [11, 20]. Examples were used to test the identification scheme for different number of parameters with higher order nonlinear plants. The results of the new scheme have demonstrated the algorithm. It is observed that the results are very close to the result reported in [1]. However, more work shall be done by other contributors to tune the weight matrix calculation such that it would produce more accurate estimates.

4. Developed an iterative frequency domain based algorithm to identify Hammerstein-Wiener models to minimize the $L_2 - gap$ error between the true model and identified model. This work based on Bai's Two Stage Identification algorithm [1], Lawson generalized Algorithm [47], and the work in Chapter 3 and 4 which as a result of the work done by both Al-Amer and Al-Sunni[11, 20]. Illustrative examples were used to test the identification scheme for different number of parameters with higher order nonlinear plants. The results were not close to the true parameters. However, after

selecting α to be the denominator of the $L_2 - gap$, the results have improved to be close to the true parameters for only the lower order nonlinear plants (< 8 parameters). Therefore, our investigations revealed that more work shall be done by other contributors to tune the weight equation such that it would produce more accurate and stable estimated parameters.

5. This $\nu - gap$ metrics was also investigated to see if we can extend the work which is done on the $L_2 - gap$ and extends it to Bai's model. However, more work is needed to be done by other contributors in this direction.

9.2 Recommendations for Future Work

Scientific research is an ongoing process and there is always some room for improvement. The following is a brief list of suggestions for possible future work by other MS students and/or contributors of work in this research area:

- In this thesis, the use of Float Genetic Algorithm (FGA) was used to identify Bai's model. Binary Generic Algorithm (BGA) optimization techniques should be tried and compared to the FGA performance
- Use improvement procedures such the Sequential Quadratic Programming (SQP) to improve the results of the GA procedure by starting at the final

point determined by the GA.

- Use Simulated Annealing (SA) algorithm as an optimization tool and compare the result with the GA's.
- Explore the development of a recursive / online (using frequencies intervals at each run) identification schemes of the new identification schemes developed in this thesis.
- Considered some very complex type of nonlinearities and for some more general type of models.
- Work on both the H_∞ and $L_2 - gap$ based identification algorithms to tune both algorithms to produce more accurate parameters estimation
- Work on $\nu - gap$ metrics to develop a $\nu - gap$ based identification algorithms of Hammerstein-Weiner nonlinear system and extend it to Bai's model.

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